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Ambiguity, Pessimism, Optimism and Financial Crises in a Simple Global Game Model

Daniel Laskar*

April 4, 2013

Abstract

We use a non-Bayesian approach to uncertainty which allows for both optimism and pessimism in a simple global game, where each signal can exhibit a bias which is ambiguous. We underline a symmetry between two models of financial crises: a liquidity crisis model, and a currency crisis model. We show that one model with pessimism becomes similar to the other model with optimism, and vice versa, which leads ambiguity to have opposite effects in the two models. We can also rationalize non-neutral effects of shifts in "market sentiment" in these models.

JEL classification: C72, D81, D82, G01

Keywords: global game, financial crises, ambiguity, optimism, pessimism, market sentiment, coordination.

1 Introduction

It is an open issue whether more uncertainty may increase financial difficulties and could trigger a financial crisis. As a framework of analysis, a "global game", where players only receive some imperfect signal on the level of "fundamentals" which can affect the possibility of a crisis, has often been used to study the role that uncertainty may play in financial crises. The reason is that, by using such a global game framework, one can get rid of the multiplicity of equilibria that could otherwise arise.

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For there are some complementarities between the actions of the participants in the financial markets. Thus, for example, a creditor will be more willing to roll over a loan for some investment project if other creditors also roll over their loans, because otherwise the investment project may encounter liquidity problems which may jeopardize its success. Depositors will have more incentives to keep their deposits at a bank if other depositors also keep their deposits, because this would make a bank failure less likely. Speculators may be more willing to attack a currency in the foreign exchange market if other speculators also attack the currency because the more massive the attack is, the more successful it can be.

It has been emphasized in the literature that such complementarities can lead to multiple equilibria of the corresponding game, at least for some intermediate range of the values of fundamentals. There could be an equilibrium without a financial crisis and an equilibrium with a financial crisis[1]. This multiplicity of equilibria arises when the state of fundamentals is common knowledge to players. However, if we consider a modified game, called a "global game", where each player only receives some imperfect signal on the state of fundamentals, this multiplicity of equilibria can be shown to disappear. And this remains true even if the noise of the signal received by each player is small[2].

In such a global game framework, where there is a unique equilibrium, comparative statics can be done in order to study the effect of uncertainty on the equilibrium. The results obtained in the literature indicate that uncertainty may or may not increase the probability of a crisis, and that the answer may depend on the parameters of the model[3].

Such an analysis of uncertainty in a global game framework was usually done under the standard Bayesian expected utility approach. However, some insufficiencies of the expected utility approach have been underlined. As Knight (1921) already emphasized, the expected utility approach may not be adequate when we are in a situation where probabilities are not known. And, as initially underlined by Ellsberg (1961), decision makers seem to exhibit an "aversion to ambiguity": they prefer a situation with known probabili-

[1] See Diamond and Dybvig (1983) and Obstfeld (1996).

[2] Global games have been introduced by Carson and van Damme (1993). Morris and Shin (2003) present a survey on global games and on some issues related to global games.

Such a global game framework has been used for the analysis of financial crises. It has been applied to currency attacks (Morris and Shin (1998)); to the situation where creditors have to decide to roll over their loans (Morris and Shin (2004)); and to the issue of bank runs (Goldstein and Pauzner (2005) and Rochet and Vives (2004)).

[3] See for example Metz (2003). See Prati and Sbracia (2010) for an empirical application to the case of episodes of currency crises.

ties to a situation with unknown probabilities. That is why in the last two or three decades, non Bayesian approaches to uncertainty have been developed in order to take into account the presence of ambiguity (or "Knightian uncertainty"). Ambiguity averse players are "pessimistic" because, for each decision considered, they give more relative weight to unfavorable events[4].

In fact, there has been a few analyses which introduce ambiguity in a global game. Ui (2009) considers a rather general global game with a continuum of players and two actions, and assumes that the probability distribution of the signal received by each player is not known: there is ambiguity on some parameter of this probability distribution. Players exhibit aversion to ambiguity and use a maxmin criterion of expected utility. Ui (2009) then applies the analysis to a model of currency crises and to a model of bank runs. It is shown that ambiguity has opposite effects in the two models: while more ambiguity increases the probability of a crisis (i.e. of a bank run) in the model of bank runs, more ambiguity decreases the probability of a crisis in the model of currency crises. Kawagoe and Ui (2010) consider a two-player version of the model of Morris and Shin (2004), which is of a global game model where creditors have to decide whether to roll over their loans for some project they are financing. The results obtained by Kawagoe and Ui (2010) imply that ambiguity increases the probability of a crisis in such a model.

As explained in Ui (2009), more ambiguity gives less incentives to choose the action which has a payoff which depends on the state of unknown fundamentals. This is why ambiguity has opposite effects in the two kinds of models. The action which has a payoff which depends on the state of fundamentals is the action which helps to prevent a crisis to occur (to roll over the loan) in the model of liquidity crises with creditors, but is the action which helps the crisis to occur (to attack the currency) in the model of currency crises..

In these analyses, more ambiguity has an effect because it leads each player to infer less favorable values of the fundamental from any signal this player receives. Laskar (2012), has underlined that there is an additional channel. Laskar (2012) introduces ambiguity in a global game model of liquidity crises (where creditors have to decide whether to roll over their loans for some project or not) but relaxes the implicit assumption, which was made in Kawagoe and Ui (2010) and Ui (2009), that the probability distributions of the signals received by the players were the same for all the players. By allowing these probability distributions to be different, Laskar (2012) shows that ambiguity lessens the amount of coordination that each player perceives

[4] Two classical references are Gilboa and Schmeidler (1989) and Schmeidler (1989). For some economic applications, see for example Mukerji and Tallon (2004).

when (s)he rolls over his/her loan. This has the effect of reducing the incentive to roll over the loan. Consequently, this new channel, going through the perceived degree of coordination, makes more ambiguity increase the probability of a crisis. This new effect reinforces the effect emphasized in Kawagoe and Ui (2010); and therefore, through these two kinds of effects, more ambiguity always make a crisis more likely in this liquidity crisis model with creditors.

As suggested in Laskar (2012), this new channel, which goes through the negative effect of ambiguity on the perceived coordination of players, should also have opposite implications in the two types of models of financial crises previously mentioned. The reason is that, in each of the two models, the action which has a payoff which benefits from coordination is also the same as the action previously mentioned (to roll over the loan or to attack the currency, respectively). Because of the opposite implications, in terms of helping a financial crisis to occur or not, that such an action has in the two models, this channel should also lead to opposite effects in the two models.

These analyses assume that decision makers are ambiguity averse. However, decision makers can sometimes exhibit "ambiguity preference" instead of ambiguity aversion, which lead them to be optimistic rather than pessimistic[5]. It would therefore be worthwhile to have the possibility for players to exhibit a behavior which can be optimistic as well as pessimistic. This seems particularly important for the analysis of financial markets. These markets have often been the subjects of waves of optimism or pessimism; and the switch from optimism to pessimism, or vice versa, could play a role in financial crises. Some of the new approaches to uncertainty which incorporate the presence of ambiguity, and therefore go beyond the standard expected utility approach, actually give a framework in which both pessimism and optimism can be taken into account (Chateauneuf et al. (2007)). In the present paper we will use such an approach to introduce both optimism and pessimism in the analysis.

Furthemore, besides what may be viewed as optimistic or pessimistic behavior, financial markets may also be the subject of fluctuating states of "market sentiment". In such a situation, participants to the financial markets may receive "biased" signals, and the amount of the bias may represent the state of market sentiment. A decrease (increase) in the bias would correspond to a downward (upward) shift in market sentiment. Cheli and Della Posta (2007) have introduced such biased signals in a global game model

[5]For some references on experimental evidence, see Eichberger and Kelsey (2009), who propose a new equilibrium concept for games under ambiguity with both optimimistic and pessimistic behavior (this is however not the approach we use here).

of currency attacks. They find that when these biases are expected by the speculators, then nothing is changed. This means that any shift in market sentiment, if expected, is without any real consequence. It is only when the speculators do not expect the correct bias, that some real effect is produced. This means that, in order to obtain some effect, it is necessary to assume that expectations are not correct. This may give a rather limited scope to such situations. The analysis of Cheli and Della Posta (2007) was however done under the usual expected utility approach. In the present paper, we will argue that, when there is ambiguity, then a shift in market sentiment (i.e. a change in the biases of the signals) may have an effect, and that such a result can be obtained without having to rely on market participants having wrong expectations: the behavior of market participants can be fully consistent with the non-Bayesian approach to uncertainty which will be used.

Therefore, in the present paper, we will consider a global game model with ambiguity which incorporates the three previous features we have underlined. First, it will be able to encompass and contrast both types of models which, in the literature, have been shown to give opposite results when we consider the effect of ambiguity on financial crises; and, in this comparison, we will explicitly allow for the two kinds of channels which have been underlined[6]. Second, we will use a non Bayesian approach to uncertainty which takes into account both optimistic and pessimistic behavior in the presence of ambiguity, and we will study the role that the degree of pessimism versus optimism may play in financial crises. Third, biased signals, with ambiguous biases, will be introduced in the analysis, which will alllow us to obtain non-neutral effects of shifts in market sentiments in a way compatible with our approach to decison under uncertainty.

In order to keep the analysis simple, we will limit our analysis to a linear global game model, often given as a simple example of a global game in the literature. This linearity of the model will prevent us from interpreting the results strictly in terms of "financial crises", and therefore we will rather consider whether there are more or less "financial difficulties".

In Section 2 we present the framework of analysis, the two kinds of models, and how these two models may be incorporated in a similar analysis. Section 3 derives the equilibrium of the game under ambiguity. In Section 4, we examine how the biases, market sentiment, ambiguity and pesimism/optimism

[6]Note that, in Laskar (2012), where the second channel going through the perceived coordination of players is present, only the model where creditors have to decide to roll over their loans was actually formally developed, and it was only a two player version. The simple linear global game model that we will use here can therefore also be viewed as some formal analysis which compares the two types of models of financial crises while taking into account both channels of the effect of ambiguity.

affect this equilibrium and the amount of financial difficulties. In Section 5 we compare the results for the two models. Section 6 concludes

2 Framework of analysis

2.1 Models

We will consider two global game models, which represent two cases of a simple global game model. Each model can be viewed as trying to take into account, in a very simple linear way, two kinds of models of financial crises. In each model there is a continuum of players, and each player can take two actions, we will denote by C (or C') and N (or N').

2.1.1 First model: creditors financing some project

The first model corrresponds to the situation where a group of players (creditors) finance some investment project through a collaterized debt. There is a continuum of players, whose indices are uniformly distributed on $[0,1]$. At an interim stage, each player has to decide whether to roll over his/her loan (action C) for the project, or to foreclose the loan and then get the value of the collateral (action N). The value of the project depends on two factors. First, it depends on the value of some variable θ, which represents the level of "fundamentals". A larger value of θ, which means stronger fundamentals, increases the value of the project. Second, the value of the project also depends on the proportion l of the players who choose to roll over their loans. When some creditors do not roll over their loans, this creates some disruption and makes the project less successful. Therefore the larger l is, the greater will be the value of the project. For simplicity, we will consider a linear function of these variables. The value of the project will be assumed to be equal to $\theta - 1 + l$. If we denote by u_i the payoff to player i, and by a_i the action this player takes, we have

$$u_i(\theta, l, a_i) = u(\theta, l, a_i) = \{ \begin{array}{c} \theta - 1 + l \text{ if } a_i = C \\ 0 \text{ if } a_i = N \end{array} \tag{1}$$

where the value of the collateral has been normalized to zero.

This model is actually identical to the simple linear example with a continuum of players given in Morris and Shin (2003)[7]

[7] This model is itself derived from the two-player example of Carlsson and van Damme (1993).

If the variable θ were common knowledge, then we would have to make the distinction beween three cases. As we have $0 \leq l \leq 1$, then, in the case $\theta < 0$, action N would be a dominant strategy; while, in the case $\theta > 1$, action C would be a dominant strategy. And in the case $0 \leq \theta \leq 1$, there would be more than one Nash equilibria: If we consider pure Nash equilibria, there would be two equilibria, one where all players choose C, and an other one where all players choose N.

However, as in the global game literature, we will assume that each player i only receives an imperfect signal x_i on θ, which is given by

$$x_i = \theta + \xi_i \tag{2}$$

where ξ_i is a random variable. We will assume that θ is drawn from a uniform distribution on $\left[\frac{1}{2} - \delta, \frac{1}{2} + \delta\right]$, where we have $\delta > 0$[8].

Usually, in the global game literature, the players are assumed to know the probability distribution of ξ_i, which is assumed to have a zero mean and some known variance σ. Then, it can be shown that, in the case $0 \leq \theta \leq 1$, the multiplicity of equilibria disappears. There is a unique equilibrium which is an equilibrium in "switching strategies", where player i takes action C if we have $x_i > k$ and takes action N if we have $x_i \leq k$. This means that a player who receives a sufficiently high signal rolls over the loan (action C). The equilibrium value of the "switching point" k can be shown to be equal to $\frac{1}{2}$ (see Morris and Shin (2003)).

In our analysis, as in Ui (2009), we will assume that the distribution of ξ_i is not known and that there is ambiguity on an underlying parameter of this distribution. However, contrary to Ui (2009) who makes the implicit assumption that all the ξ_i have the same probability distribution[9], we will assume, as in Laskar (2012), that these probability distributions may be different from one player to the other. For players may receive different kinds of signals, from different sources, get advices from different experts, or weight them differently. This will change not only the signal received but also the probability distribution of the signal. As underlined in Laskar

[8] In Morris and Shin (2003), θ is assumed to be randomly drawn from the real line. Here, we will not use such an improper prior because this would actually make the effects of the exogenous parameters on financial difficulties negligible. We want however to keep the prior on θ non informative when a player receives a signal x_i equal to the equilibrium switching point k^*. This will require a large enough value of δ (see Section 3.2 and Appendix 1).

[9] There are some other differences. First, Ui (2009), when presenting this example, makes the assumption that both θ and ξ_i follow a normal distribution. Second, the parameter on which there is ambiguity is the variance of the distribution (or equivalently the precision of the signal), while the mean is taken equal to zero. Here, the variance is assumed to be known, and we take the mean as the parameter which is ambiguous.

(2012), this adds an other channel through which ambiguity may affect the equilibrium of the game. This additional channel goes through the expected coordination of players.

As in Laskar (2012), we will consider that there is ambiguity on the mean of the distribution of ξ_i. Taking the mean as the ambiguous parameter of the distribution of ξ_i has the advantage of simplicity, but it will also allow us to raise the issue of shifts in "market sentiment", which can be represented by a shift in the mean, i.e. the "bias", of the signal. This may be an important issue in the study of financial markets and crises. This issue has been considered in a global game model of currency crises by Cheli and Della Posta (2007) under a usual expected utility approach. But, as we have indicated in the Introduction, our analysis under ambiguity will permit us to obtain non-neutral effects of market sentiment which are fully compatible with the rationality of the approach to uncertainty which is taken. We therefore have

$$\xi_i = \mu_i + \varepsilon_i \tag{3}$$

where the variables ε_i, ε_j, $i \neq j$, and θ are assumed to be mutually stochastically independent.. All the ε_i follow the same known zero-mean probability distribution. We will also assume that ε_i has a finite support $[-\rho, \rho]$, with $\rho > 0$. Therefore we get

$$x_i = \theta + \mu_i + \varepsilon_i \tag{4}$$

Thus each player receives a signal which may exhibit some bias μ_i, and this bias depends on i. We may have $\mu_i \neq \mu_j$, which means that the biases are not necessarily the same for all the players. There is ambiguity on the value of each bias μ_i, and, in section 2.2.1 below we will consider how this ambiguity is taken into account.

2.1.2 Second model: speculators attacking a currency

The second model tries to represent currency attacks by a group of speculators, in a linear way, as in the previous model. Each player (speculator) has to decide to attack a currency (action C') or not to attack (action N'). (In both models, we denote by "C (or C')" the action which has a payoff which benefits from the "coordination" of players. Furthermore, in both models, the payoff to action C (or C') depends on the unkown level of fundamentals θ (or θ'), while the payoff to action N (or N') is constant (and always equal to zero).) As in the first model, the success of the attack against the currency depends on the same kind of two factors. First, it depends on the level θ' of fundamentals (we put primes on variables or parameters of the second model). Here, as in the first model, a greater value of θ' means "stronger"

fundamentals. Thus, an increase in θ' would make the government more willing to resist the attack by defending the currency; and, furthemore, for a given willingness of the the government to defend the currency, the amount of depreciation of the currency will be smaller with stronger fundamentals. These two effects make the payoff to attacking the currency smaller when fundamentals get stronger. Second, the payoff to attacking the currency will also increase with the proportion l' of speculators who attack the currency because this will make the attack more successful. Taking also a linear function of these two variables, the payoff u_i' of a player (speculator) i is

$$u_i'(\theta', l', a_i') = u'(\theta', l', a_i') = \{ \begin{array}{c} l' - \theta' \text{ if } a_i' = C' \\ 0 \text{ if } a_i' = N' \end{array} \tag{5}$$

As previously, we assume that each speculator receives some imperfect signal x_i'. We make similar asssumptions, and therefore equations (2), (3) and (4) hold for the corresponding variables x_i', θ', ξ_i', ε_i' and μ_i'. As in the first model, there is ambiguity on the biases μ_i' of the signals.

2.2 Criterion under ambiguity and equilibrium condition

2.2.1 Criterion

In this section we will only consider the first model. In section 2.3 below, we will consider the second model and examine how we can go from the equilibrium solution of the first model to the equilibrium solution of the second model.

We have assumed that the biases μ_i of the signals are ambiguous. As indicated in the Introduction, we will use a non Bayesian approach to decision under uncertainty which can take into account this ambiguity and which, furhermore, allows players to be pessimistic as well as optimistic when facing this ambiguity.

We will assume that each μ_j belongs to the interval $\left[\underline{\mu}, \overline{\mu}\right]$, which represents the possible values for μ_j. Let $M \equiv \left\{\mu_j; j \in [0,1]\right\}$ be the set of the given biases. Let $\mathcal{M}$ the set of M such that $\mu_j \in \left[\underline{\mu}, \overline{\mu}\right]$ for all $j \in [0,1]$.

We will look for an equilibrium in switching strategies where, as previously indicated, player i takes action C if we have $x_i > k$ and takes action N if we have $x_i \leq k$. Let $\pi_M(k, x_i, a_i)$ be the expected utility of player i, conditional on having received the signal x_i, when this player takes action a_i and all other players $j \neq i$ follow the switching strategy with switching point k, and when the set of biases M is given. We have

$$\pi_M(k, x_i, a_i) \equiv E_M(u(\theta, l, a_i) \mid x_i) \tag{6}$$

where we denote by E_M the expectation with respect to the probability distribution obtained when the biases are M. From the definition of l given above, using (4), we can write[10]

$$l = \int_0^1 1_{\{\theta+\mu_j+\varepsilon_j>k\}} dj \tag{7}$$

where $1_{\{\theta+\mu_j+\varepsilon_j>k\}} = 1$ if $\theta + \mu_j + \varepsilon_j > k$, and $1_{\{\theta+\mu_j+\varepsilon_j>k\}} = 0$ otherwise.

For any given switching point k, each player i, who receives the signal x_i, will be assumed to choose a_i which maximizes $\Omega_{\alpha,i}(k, x_i, a_i)$ given by[11]

$$\Omega_{\alpha,i}(k, x_i, a_i) = \alpha \min_{M\in\mathcal{M}} \pi_M(k, x_i, a_i) + (1-\alpha) \max_{M\in\mathcal{M}} \pi_M(k, x_i, a_i) \tag{8}$$

There are two terms in (8), and $\Omega_{\alpha,i}(k, x_i, a_i)$ is a weighted average of these two terms. The first, which is $\min_{M\in\mathcal{M}} \pi_M(k, x_i, a_i)$, gives, for each action a_i, the expected utility in the worst case and therefore comes from a pessimistic behavior. The second term, which is $\max_{M\in\mathcal{M}} \pi_M(k, x_i, a_i)$, gives the expected utility in the best case and is therefore due to optimism.

Parameter α, which determines the relative weight given to these two terms, is an index of pessimism (and $1-\alpha$ is an index of optimism). In the case $\alpha > \frac{1}{2}$ there is more pessimism than optimism, while in the case $\alpha < \frac{1}{2}$ there is more optimism than pessimism.

Consider parameter η defined by

$$\eta \equiv \frac{1}{2}(\overline{\mu} - \underline{\mu}) \tag{9}$$

[10] As there is a continuum of players, what happens for player i does not matter in the integral.

[11] Such a criterion can be derived from a Choquet expected utility approach to uncertainty by taking the case of "neo-additive capacities" (see Chateauneuf et al. (2007) who also provide behavioral axioms). This actually leads to a criterion which evaluates any function f according to the multi-prior form $\alpha \min_{p\in\mathcal{D}} \int f dp + (1-\alpha) \max_{p\in\mathcal{D}} \int f dp$ (see Chateauneuf et al. (2007) eq. (1) p. 543)), where p is a probability distribution. The set of probabilities $\mathcal{D}$ represents ambiguity, and α represents the degree of pessimism (and $1-\alpha$ the degree of optimism).

In our analysis we consider the special case where $\mathcal{D}$ is the set of probability distributions having their supports included in $[\underline{\mu}, \overline{\mu}]$. In Chateauneuf et al. (2007) (see p.541), this special case can be obtained by giving a zero weight to the probability distribution of reference in the definition of the neo-additive capacity, and by taking the "set of null events" as the set given by the values of M which do not belong to $\mathcal{M}$. Thus, $[\underline{\mu}, \overline{\mu}]$ can be considered as characterizing ambiguity in our analysis. Note also that, in (8), we have taken the maximum or minimum for all $M \in \mathcal{M}$ rather than for all the probability distributions p belonging to $\mathcal{D}$, because this actually gives the same results in our model. The worst case and the best case for these probabilities would yield the probability distributions which give probablity 1 to the values of M obtained in the worst case and the best case of Proposition 2 below.

A greater value of η represent more ambiguity on the possible values of the biases. In the special case $\eta = 0$ of no ambiguity, we would obtain the usual expected utility criterion.

2.2.2 Equilibrium condition

For a given k, each player i chooses his/her action a_i which maximizes $\Omega_{\alpha,i}(k, x_i, a_i)$. Let $b(k)$ be the value of the signal x_i which makes player i indifferent between choosing action C or action N. The value $b(k)$ satisfies the equation $\Omega_{\alpha,i}(k, b(k), C) = \Omega_{\alpha,i}(k, b(k), N)$, which gives $\Omega_{\alpha,i}(k, b(k), C) = 0$. As $\Omega_{\alpha,i}(k, x_i, a_i)$ is an increasing function of x_i[12], player i chooses C if we have $x_i > b(k)$ and chooses N if we have $x_i \leq b(k)$[13]. Therefore, when other players follow the switching strategy with switching point k, player i follows the switching strategy with switching point $b(k)$. As a consequence, if k satisfies the equation $b(k) = k$, the switching strategy with switching point k is an equilibrium of the game. The equilibrium value of k is therefore given by the equation $\Omega_{\alpha,i}(k, k, C) = 0$.

2.3 Link between the two models

We want to see how we can go from the equilibrium solution of the first model to the equilibrium solution of the second model. The primary aim of our analysis will be to examine what are the effects of the exogenous parameters of the models on financial difficulties. In the first model, financial difficulties are smaller when the proportion l of players (creditors) who take action C, i.e. who roll over their loans, is greater. On the contrary, in the second model, financial difficulties are smaller when the proportion of player who choose N' (and not C'), i.e. who do not attack the currency, is greater. This proportion is equal to $h' \equiv 1 - l'$. Replacing l' by $1 - h'$ in (5), we can write the utility function of player i, in the second model, as a function of h' instead of l'. We get

$$\widehat{u}_i'(\theta', h', a_i') = \widehat{u}'(\theta', h', a_i') = \{ \begin{array}{c} 1 - h' - \theta' \text{ if } a_i' = C \\ 0 \text{ if } a_i' = N \end{array} \tag{10}$$

Comparing (1) and (10), we simply get opposite utility functions in the two models: $\widehat{u}_i' = \widehat{u}' = -u_i = -u$. Now, in the second model, consider the

[12] A greater signal x_i shifts the probability distribution of θ conditional on x_i towards greater values of θ. This increases both the expected value of θ and the expected value of l. This implies that $\pi_M(k, x_i, a_i)$ and therefore also $\Omega_{\alpha,i}(k, x_i, a_i)$ are increased.

[13] It is assumed that if a player is indifferent between C and N, (s)he chooses N. This is without any real consequences.

switching strategy where player i takes action N' (and not C' as in the first model) if we have $x_i' > k'$ and takes action C' if we have $x_i \leq k$. As h' is the proportion of players who take action N', h' satisfies the same equality, given by (7), as l satisfies in the first model:

$$h' = \int_0^1 1_{\{\theta' + \mu_j' + \varepsilon_j' > k'\}} dj \tag{11}$$

Then, let $\widehat{\pi}'_{M'}(k', x_i', a_i')$ be defined in the same way as $\pi_M(k, x_i, a_i)$ in the first model, by (6). We have

$$\widehat{\pi}'_{M'}(k', x_i', a_i') \equiv E_{M'}(\widehat{u}'(\theta', h', a_i') \mid x_i) \tag{12}$$

As we have $\widehat{u}' = -u$, and as h' satisfies the same kind of equality (11) as the equality (6) that satifies l, we get

$$\widehat{\pi}'_{M'}(k', x_i', a_i') = -\pi_{M'}(k', x_i', a_i') \tag{13}$$

The corresponding criterion $\widehat{\Omega}'_{\alpha',i}(k', x_i', C')$ is given by the same equality as (8):

$$\widehat{\Omega}'_{\alpha',i}(k', x_i', C') = \alpha' \min_{M' \in \mathcal{M}'} \widehat{\pi}'_{M'}(k', x_i', a_i') + (1 - \alpha') \max_{M' \in \mathcal{M}'} \widehat{\pi}'_{M'}(k', x_i', a_i') \tag{14}$$

From (13), and $\min_{M' \in \mathcal{M}}(-\pi_{M'}(k', x_i', a_i')) = -\max_{M' \in \mathcal{M}}(\pi_{M'}(k', x_i', a_i'))$ and $\max_{M' \in \mathcal{M}}(-\pi_{M'}(k', x_i', a_i')) = -\min_{M' \in \mathcal{M}} \pi_{M'}(k', x_i', a_i')$, we get

$$\widehat{\Omega}'_{\alpha',i}(k', x_i', C') = -\Omega_{1-\alpha',i}(k', x_i', C') \tag{15}$$

This means that, in order to obtain $\widehat{\Omega}'_{\alpha',i}(k', x_i', C')$, we first replace α' by $1 - \alpha'$ in $\Omega_{\alpha',i}(k', x_i', C')$ given by (8), and, second, we take the opposite value. This last change is without any consequence for the equilibrium condition because writing $\widehat{\Omega}'_{\alpha',i}(k', x_i', C') = 0$ is equivalent to writing $-\widehat{\Omega}'_{\alpha',i}(k', x_i', C') = 0$.

We therefore have the following result[14]:

[14] An other obvious way to link the two models would be to introduce the variable $\widetilde{\theta}' \equiv 1 - \theta'$, where an increase in $\widetilde{\theta}'$ would mean weaker fundamentals (although better fundamentals from the point of view of speculators). Then, we would get the same utility functions in the two models. But this means that we should then interpret in an opposite way the corresponding changes in the biases $\widetilde{\mu}_i'$ or in the possible baises $\underline{\widetilde{\mu}}'$ or $\overline{\widetilde{\mu}}'$; and, while an increase in l would correspond to smaller financial diffficulties, an increase in l' would mean greater financial difficulties. We have preferred to introduce the variable $h' \equiv 1 - l'$ and to use the symmetry between optimism and pessimism.

Proposition 1 *If we want to know the equilibrium switching point or the implications in terms of financial difficulties, the only difference between the two models is that the role played by pessimism in one model is the same as the role played by optimism in the other model, and vice versa (we have to replace α by $1-\alpha'$).*

As we can go in a straightforward way from the first model to the second model, in the next two sections we will develop the analysis and the results only for the first model. Then, in Section 5 we will consider what these results imply for the second model and compare the results for the two models.

3 Equilibrium

3.1 Worst case and best case

When a_i is equal to N, $u_i(\theta, l, N)$ is always equal to zero, and therefore $\Omega_{\alpha,i}(k, x_{i,} N)$ is always equal to zero and does not depend on M. When a_i equal to C, then, from (1), we have to consider the corresponding expected values of θ and l. Then, from (1) and (6), we get

$$\pi_M(k, x_i, C) = E_M(\theta \mid x_i) + E_M(l \mid x_i) - 1 \tag{16}$$

We first have the following result:

Proposition 2 *1. We have* $\min_{M\in\mathcal{M}} \pi_M(k, x_i, C) = \pi_{M_{Wi}}(k, x_i, C)$, *where the worst case M_{Wi} for player i is given by $\mu_i = \overline{\mu}$ and $\mu_j = \underline{\mu}, j \neq i$.*

2. We have $\max_{M\in\mathcal{M}} \pi_M(k, x_i, C) = \pi_{M_{Bi}}(k, x_i, C)$, *where the best case M_{Bi} for player i is given by $\mu_i = \underline{\mu}$ and $\mu_j = \overline{\mu}, j \neq i$.*

Proof: First consider the choice of the μ_j, $j \neq i$. These biases enter $\pi_M(k, x_i, C)$ only through the term $E_M(l \mid x_i)$ because they do not affect the signal x_i and therefore do not affect the distribution of θ conditional on x_i. For any j, consider M_1 and M_2 which differ only by the value of μ_j. If we have $\mu_{2j} < \mu_{1j}$, then (7) implies $l_2 \leq l_1$. As a consequence the values of μ_j, $j \neq i$ which minimize $E_M(l \mid x_i)$ (and therefore also $\pi_M(k, x_i, C)$) are given by $\mu_j = \underline{\mu}$ for all j (and the values that maximize $E_M(l \mid x_i)$ are given by $\mu_j = \overline{\mu}$ for all j).

Now consider the choice of μ_i. As there is a continuum of players, player i 's action has a negligible impact on l: In the integral which appears in (7) the previous kind of effect of μ_i is negligible. Therefore the effect of μ_i only goes through its effect on the conditional distribution of θ. From (4), we have

$\theta = x_i - \mu_i - \varepsilon_i$. Consequently, if μ_i is increased, the probability distribution of θ conditional on the signal x_i, is shifted toward lower values of θ. This decreases $E_M(\theta \mid x_i)$. It also decreases $E_M(l \mid x_i)$ because when θ is lower the set $\{\theta + \mu_j + \varepsilon_j > k\}$ is also smaller and therefore, from (7), l is smaller. This implies that the minimum of $\pi_M(k, x_i, C)$ is obtained for $\mu_i = \overline{\mu}$ (and the maximum is obtained for $\mu_i = \underline{\mu}$). QED

Proposition 2 highlights that, as in Laskar (2012), the assumption that the probability distributions of the signals may be different, i.e. that we may have $\mu_i \neq \mu_j$, may play an important role when there is ambiguity. For, from the point of view of any player i, both the worst case and the best case require that μ_i and $\mu_j, j \neq i$ take different values. In fact, they take the most possible distant values: when μ_i takes one of the extreme values $\overline{\mu}$ (or $\underline{\mu}$), then μ_j takes the other extreme value $\underline{\mu}$ (or $\overline{\mu}$).

3.2 Equilibrium value of k

As we have seen, the equilibrium condition is $\Omega_{\alpha,i}(k, k, C) = 0$. From (8), (16) and Proposition 2, we have

$$\begin{aligned} \Omega_{\alpha,i}(k,k,C) &= \alpha E_{M_{Wi}}(\theta \mid x_i = k) + (1-\alpha) E_{M_{Bi}}(\theta \mid x_i = k) \\ &\quad + \alpha E_{M_{Wi}}(l \mid x_i = k) + (1-\alpha) E_{M_{Bi}}(l \mid x_i = k) - 1 \end{aligned} \tag{17}$$

Consider the mean possible bias μ_0 defined by

$$\mu_0 \equiv \frac{1}{2}\left(\overline{\mu} + \underline{\mu}\right) \tag{18}$$

From (9) and (18), we have

$$\overline{\mu} = \mu_0 + \eta; \underline{\mu} = \mu_0 - \eta \tag{19}$$

From (4), the condition $x_i = k$ can be written

$$\theta = k - \mu_i - \varepsilon_i \tag{20}$$

We will assume that when a player receives a signal x_i equal to k, the prior on θ is non informative, which means that the conditional distribution of θ is then simply given by equation (20) where ε_i follows its unconditional distribution. It can be shown that the non informativeness of the prior on θ when a player receives a signal equal to the equilibrium switching point requires that parameter δ, which measures the length of the support of the uniform prior on θ, is large enough. The precise condition is given in Appendix 1. We will therefore assume that this condition is satisfied

Consider the first two terms in (17), which come from the conditional expected values of θ. From Proposition 2, (19) and (20), we get

$$\alpha E_{M_{Wi}}(\theta \mid x_i = k) + (1-\alpha) E_{M_{Bi}}(\theta \mid x_i = k) = k - \mu_0 - (2\alpha - 1)\eta \quad (21)$$

Now consider the last two terms in (17), which come from the conditional expected values of l. From (7), we have

$$E_M(l \mid x_i = k) = \int_0^1 \Pr\left(\theta + \mu_j + \varepsilon_j > k\right) dj \quad (22)$$

Using (20), this gives

$$E_M(l \mid x_i = k) = \int_0^1 \Pr\left(\varepsilon_j - \varepsilon_i > \mu_i - \mu_j\right) dj \quad (23)$$

From Proposition 2 and (9), in the worst case we have $\mu_i - \mu_j = 2\eta$, and in the best case we have $\mu_i - \mu_j = -2\eta$. This gives

$$E_{M_{Wi}}(l \mid x_i = k) = \Pr(\varepsilon_j - \varepsilon_i > 2\eta) \quad (24)$$

$$E_{M_{Bi}}(l \mid x_i = k) = \Pr(\varepsilon_j - \varepsilon_i > -2\eta) \quad (25)$$

Let $\psi(.)$ be the c.d.f. of the distribution of $\varepsilon_j - \varepsilon_i$ (i.e. we have $\psi(z) = \Pr(\varepsilon_j - \varepsilon_i \leq z)$). As the distribution of $\varepsilon_j - \varepsilon_i$ is symmetric we have $1 - \psi(-2\eta) = \psi(2\eta)$. From (24) and (25), we have $E_{M_{Wi}}(l \mid x_i = k) = 1 - \psi(2\eta)$ and $E_{M_{Bi}}(l \mid x_i = k) = \psi(2\eta)$, which gives

$$\alpha E_{M_{Wi}}(l \mid x_i = k) + (1-\alpha) E_{M_{Bi}}(l \mid x_i = k) = \frac{1}{2} - (2\alpha - 1)\left[\psi(2\eta) - \frac{1}{2}\right] \quad (26)$$

From (17), (21) and (26) we get

$$\Omega_{\alpha,i}(k, k, C) = k - \frac{1}{2} - \mu_0 - (2\alpha - 1)\left[\eta + \psi(2\eta) - \frac{1}{2}\right] \quad (27)$$

The equilibrium value k^*, which is given by $\Omega_i(k, k, C) = 0$, is therefore

$$k^* = \frac{1}{2} + \mu_0 + (2\alpha - 1)\left[\eta + \psi(2\eta) - \frac{1}{2}\right] \quad (28)$$

4 Effects on financial difficulties

We want to examine how the parameters of the model $\{M, \mu_0, \eta, \alpha\}$ affect the amount of "financial difficulties". These parameters include the true values of the biases $M \equiv \{(\mu_j),\ j \in [0,1]\}$,the mean value μ_0 of the possible biases, the ambiguity parameter η, and the degree of pessimism α.

The smaller the number of players l who roll over their loans, the greater are the financial difficulties. Therefore, its expected value El^* at the equilibrium may be considered as an inverse index of the amount of financial difficulties: the lower this expected value is, the greater are the financial difficulties. From (7) ,we have

$$El^* = \int_0^1 \Pr\left(\theta + \varepsilon_j > k^* - \mu_j\right) dj \tag{29}$$

Using (28) , this gives

$$El^* = \int_0^1 \Pr\left(\theta + \varepsilon_j > g\left(\mu_j, \mu_0, \eta, \alpha\right)\right) dj \tag{30}$$

where $g\left(\mu_j, \mu_0, \eta\right)$ is given by

$$g\left(\mu_j, \mu_0, \eta, \alpha\right) = \frac{1}{2} - \left(\mu_j - \mu_0\right) + (2\alpha - 1)\left[\eta + \psi(2\eta) - \frac{1}{2}\right] \tag{31}$$

As El^* is a decreasing function of $g\left(\mu_j, \mu_0, \eta, \alpha\right)$, for any $j \in [0,1]$, financial difficulties are an increasing function of $g\left(\mu_j, \mu_0, \eta, \alpha\right)$. Therefore, we will have to consider how the parameters of the model affect $g\left(\mu_j, \mu_0, \eta, \alpha\right)$ for any $j \in [0,1]$.

4.1 Effects of the true biases and of the mean possible bias; effect of market sentiment

First we will consider the effects of the true values of the biases $M \equiv \{(\mu_j),\ j \in [0,1]\}$ and of the mean possible bias μ_0. From (31) , $g\left(\mu_j, \mu_0, \eta, \alpha\right)$ is a decreasing function of the difference $\mu_j - \mu_0$, for any $j \in [0,1]$. This means that an increase in any value of $\mu_j - \mu_0$ reduces financial difficulties. Consequently, an increase in any true bias μ_j lessens financial difficulties; and, on the contrary, an increase in the mean possible bias μ_0 raises financial difficulties. The reasons are the following. First, when μ_j increases, player j receives a more favorable (a higher) signal, which makes this player more likely to roll over the loan. Second, when the mean possible bias μ_0 increases, while

the amount of ambiguity η stays unchanged this implies that both $\underline{\mu}$ and $\overline{\mu}$ increase by the same amount. Therefore, both in the best case and in the worst case, each player substracts a greater bias to the signals (s)he receives. In both cases, the conditional probability distribution of θ is therefore shifted toward lower values. This makes each player less likely to roll over the loan.

Now, let us consider a simultaneous increase of each μ_j by some equal amount, with a corresponding equal increase of μ_0 by the same amount. This leaves $\mu_j - \mu_0$ unchanged, and, consequently, also leaves $g\left(\mu_j, \mu_0, \eta, \alpha\right)$ unchanged. Such a change would therefore have no effect on financial difficulties. The reason is that the (identical) changes in the true values of the biases μ_j would be entirely taken into account by the players through the equal corresponding change in the mean possible bias μ_0. The players (both in the worst case and in the best case) would then simply discount their signals correspondingly by the new average bias. For each player, the conditional distribution of θ would then be unchanged, and therefore nothing would be modified in the analysis.

The case where all the μ_j and μ_0 change by the same amount would correspond, under the usual Bayesian expected utility approach, to the rational expectation case where players rationally take into account the changes in the true biases[15]. And, as was underlined by Cheli and Della Posta (2007), there would be no effect on crises[16]. This would correspond to a shift in "market sentiment" which is entirely perceived by the players. There are some effects in changes in market sentiments only if we assume that the expectations of the biases held by the players may be different from the true biases. While, in the analysis of Cheli and Della Posta (2007), which is done under the usual expected utility approach, this would involve some non rationality of expectations, in our analysis this is compatible with rationality, once a non-Bayesian approach with ambiguity is taken. For, under the present non-Bayesian approach, it is now possible to preserve rationality if we consider a separate change in the biases μ_j, while simultaneously keeping unchanged the expectations of the players in the worst and best cases (by keping $\underline{\mu}$ and $\overline{\mu}$ unchanged through an unchanged μ_0 and η); or if we consider a separate change in the expectations in the players (due to an equal change in $\underline{\mu}$ and $\overline{\mu}$, through a change in μ_0 for a given η) without a corresponding change in the true biases $M \equiv \{\left(\mu_j\right), j \in [0,1]\}$. For, if we go from situation 1 given

[15] In our model, this Bayesian case would be obtained by taking $\mu_j = \underline{\mu} = \overline{\mu} = \mu_0$ for all j.

[16] Cheli and Della Posta (2007) actually consider a global game model of currency attacks. But, as we will underline in Section 5 below, the results we obtain in our first model also hold in our second model (which is a linear version of a currency attack model).

by $\left(M_1, \left(\underline{\mu}_1, \overline{\mu}_1\right)\right)$ to situation 2 given by $\left(M_2, \left(\underline{\mu}_2, \overline{\mu}_2\right)\right)$, we only have to assume that we have $\mu_{j,1} \in \left[\underline{\mu}_1, \overline{\mu}_1\right]$ and $\mu_{j,2} \in \left[\underline{\mu}_2, \overline{\mu}_2\right]$ for all j. As long as these conditions are satisfied, this would be compatible with separate changes in the μ_j or in the $(\underline{\mu}, \overline{\mu})$, where each group of parameters can be modified independently. Therefore, this first part of our analysis, which concerns the effects of the μ_j and μ_0, may be viewed as a way to make rational non-neutral effects of market sentiments

4.2 Effects of ambiguity

We consider that $\left(\underline{\mu}_2, \overline{\mu}_2\right)$ is more ambiguous than $\left(\underline{\mu}_1, \overline{\mu}_1\right)$ if we have $\overline{\mu}_2 \geq \overline{\mu}_1$ and $\underline{\mu}_2 \leq \underline{\mu}_1$ with at least one strict inequality.

4.2.1 Effect of a symmetic increase in ambiguity

First, consider the effect of a symmetric increase in ambiguity, where there is an increase in $\overline{\mu}$ with an equal decrease in $\underline{\mu}$. From (9) and (18), such a change in ambiguity corresponds to an increase in the ambiguity parameter η, holding μ_0 (and all other parameters of the model) constant. From (31), we have

$$\frac{\partial g\left(\mu_j, \mu_0, \eta, \alpha\right)}{\partial \eta} = (2\alpha - 1)\left[1 + 2f\left(2\eta\right)\right] \tag{32}$$

where the function $f(.)$ is the derivative of the function $\psi(.)$ and is therefore the density of the distribution of $\varepsilon_j - \varepsilon_i$ (because $\psi(.)$ is the c.d.f. of $\varepsilon_j - \varepsilon_i$). As we have $f(2\eta) \geq 0$, we obtain that $\frac{\partial g\left(\mu_j, \mu_0, \eta, \alpha\right)}{\partial \eta}$ has the sign of $2\alpha - 1$. This implies, that more ambiguity increases financial difficulties in the case where pessimism dominates ($\alpha > \frac{1}{2}$) and decreases financial difficulties in the case where optimism dominates ($\alpha < \frac{1}{2}$); while in the neutral case ($\alpha = \frac{1}{2}$) financial difficulties are unchanged.

In (32), there are two terms. The first is equal to $2\alpha - 1$ and comes from the derivative of the weighted averages of the expected values of θ in the worst and best cases $\alpha E_{M_{Wi}}(\theta \mid x_i = k) + (1 - \alpha) E_{M_{Bi}}(\theta \mid x_i = k)$, given by (21). This term is due to the correction that, in order to deduce the corresponding expected values of θ, each player j makes by substracting, to the signal x_j received, the biases $\overline{\mu}$ and $\underline{\mu}$ in the worst case and best case, respectively. When there is more pessimism, the increase in $\overline{\mu}$ in the worst case dominates. The increase in this bias implies more correction and therefore a lower expected value for θ in the worst case, and therefore less favorable expected fundamentals. This increases financial difficulties through

a higher equilibrium switching point, and therefore a lower probability that any player rolls over the loan. The opposite is true when there is more optimism than pessimism. Then it is the decrease in the bias $\underline{\mu}$ in the best case which dominates. As a decrease in the bias implies less correction for a given signal, this yields better expected fundamentals, i.e. a higher expected value for θ in the best case. This reduces financial difficulties through a lower equilibrium switching point

The second term in (32), equal to $(2\alpha - 1)\, 2f\,(2\eta)$, comes from the conditional expected weighted average $\alpha E_{M_{W_i}}\,(l \mid x_i = k) + (1-\alpha)\, E_{M_{B_i}}\,(l \mid x_i = k)$ in the worst and best cases, which is given by equation (26) , of the proportion l of players who roll over their loans.This term is entirely due to the fact that, as in Laskar (2012), we have allowed μ_i to be different from μ_j for $i \neq j$, Otherwise, if, as it was done in the other analyses of ambiguity in global games found in the literature (Kawagoe and Ui (2010) and Ui (2009)), we had a priori assumed that the distributions of the signals were the same for all the players, and therefore that we a priori had $\mu_i = \mu_{j,}$ then, from (23) and the fact that the distribution of $\varepsilon_j - \varepsilon_i$ is symmetric, the conditional expected value of l would always equal to $\frac{1}{2}$, and this channel of the effect of ambiguity would disappear. As, from Proposition 1, in the worst case as well as in the best case, the values of μ_i and μ_j take opposite extreme values ($\overline{\mu}$ and $\underline{\mu}$, or $\underline{\mu}$ and $\overline{\mu}$, in the worst and best cases respectively) this makes the conditional expected value of l generally different from $\frac{1}{2}$.

Thus, from (24) and (25) , we see that, when there is ambiguity, pessimism makes the conditional expected value of l smaller than $\frac{1}{2}$, and optimism makes this expected value greater than $\frac{1}{2}$. In other words, when a player considers rolling over the loan, (s)he expects a degree of coordination between his/her action and the actions of the other players which is lower than $\frac{1}{2}$ when (s)he is pessimistic, and greater than $\frac{1}{2}$ when (s)he is pessimistic. This means that the property of "Laplacian beliefs" underlined in the literature under the usual Bayesian expected utility approach is not valid anymore when there is ambiguity[17]. This literature had emphasized that, when the prior on θ is not informative, then, when a player receives a signal equal to the equilibrium

[17] As explained before, because of their implicit assumption that the probability distribuitions of the signals are a priori the same for all the players, Laplacian beliefs would also be present in the analyses of Kawagoe and Ui (2010) and Ui (2009), which also means that the channel of ambiguity going through the effect on expected coordination does not exist in their analyses.

In Laskar (2012), where ambiguity and pessimism are introduced in a two player global game of liquidity crises with creditors, this assumption is relaxed and, consequently, the two player counterpart of this Laplacian property does not hold : the conditional expected value of the proportion of other players who roll over the loan (which is equal to the probability that the other player rolls over the loan) is shown to be less than $\frac{1}{2}$.

switching point, each player believes that the proportion l of other players[18] who choose the same action is uniformely distributed on $[0,1]$ (see Morris and Shin (2003)), which implies that its conditional expected value is always equal to $\frac{1}{2}$. In our analysis, this is not true anymore: When we consider the weighted average of the expected values of l in the worst and best cases, which is what matters in the criterion under ambiguity used, this weighted average is generally different from $\frac{1}{2}$, and, as (26) indicates, depends on the level of ambiguity η. It is only in the very special case $\alpha = \frac{1}{2}$, where there is as much optimism as pessimism, that this weighted average would always be equal to $\frac{1}{2}$.

Thus, in the general case $\alpha \neq \frac{1}{2}$, the level of ambiguity η affects the relevant expected degree of coordination between players when they roll over their loans. When pessimism dominates, more ambiguity decreases this expected degre of coordination, which reduces the incentive to roll over the loan and, consequently, increases financial difficulties (through the higher equilibrium switching point that this implies). When optimism dominates, we have the opposite result.

4.2.2 Effect of an asymmetric increase in ambiguity

Ambiguity may also increase in an asymmetric way. The increase in $\overline{\mu}$ may be diffferent from the decrease in $\underline{\mu}$. In this case there is both a change in η and μ_0, and we have to add these two effects which have been previously studied. Let us consider the two extreme possibilities of an asymmetric change in ambiguity. The first corresponds to an increase in $\overline{\mu}$ with an unchanged $\underline{\mu}$ and the second to a decrease in $\underline{\mu}$ with an unchanged $\overline{\mu}$. In the first case we have a situation where it becomes possible for the bias to be higher; while in the second case, it becomes possible for the bias to be lower. We keep the true values of the biases $\{(\mu_j),\, j \in [0,1]\}$ unchanged.

Consider the first situation of a marginal increase in $\overline{\mu}$ equal to $d\overline{\mu} > 0$. From (9) and (18), this leads to an increase in both μ_0 and η equal to $\frac{d\overline{\mu}}{2}$. From (31), this implies

$$dg\left(\mu_j, \mu_0, \eta, \alpha\right) = [1 + (2\alpha - 1)(1 + 2f(2\eta))]\frac{d\overline{\mu}}{2} \tag{33}$$

From (33), financial difficulties are increased if and only if we have $\alpha > \widetilde{\alpha}$ where $\widetilde{\alpha}$ is the solution of the equation $1 + (2\alpha - 1)(1 + 2f(2\eta)) = 0$, which

[18] In the case of a continuum of players, which is the case we consider in our analysis, each player has a negligible weight, and therefore this proportion of other players is equal to the proportion l of all the players.

gives

$$\widetilde{\alpha} = \frac{f(2\eta)}{1 + 2f(2\eta)} \tag{34}$$

From (34), we have $0 < \widetilde{\alpha} < \frac{1}{2}$. Therefore, it is only in the case $\alpha < \widetilde{\alpha}$, i.e. when pessimism is sufficiently low, that financial difficulties are lessened. This condition is more severe than the condition $\alpha < \frac{1}{2}$ obtained in the case of a symmetric increase in ambiguity (due to an increase in η). The reason is that, to the effect of a greater η, we have to add the effect of a greater value of μ_0, which, as we have previously shown, increases financial difficulties.

Now consider the other opposite case of a marginal decrease in $\underline{\mu}$. We have therefore consider $d\underline{\mu} < 0$, which, from (9) and (18), implies $d\eta = -d\mu_0 = -\frac{d\underline{\mu}}{2}$. From (31), this implies

$$dg\left(\mu_j, \mu_0, \eta, \alpha\right) = \left[-1 + (2\alpha - 1)(1 + 2f(2\eta))\right]\left(-\frac{d\underline{\mu}}{2}\right) \tag{35}$$

The value of α which solves $-1 + (2\alpha - 1)(1 + 2f(2\eta)) = 0$ is actually equal to $1 - \widetilde{\alpha}$. Therefore a decrease in $\underline{\mu}$ increases financial difficulties in the case $\alpha > 1 - \widetilde{\alpha}$, and reduces them in the case $\alpha < 1 - \widetilde{\alpha}$. As we have $0 < \widetilde{\alpha} < \frac{1}{2}$, this implies $\frac{1}{2} < 1 - \widetilde{\alpha} < 1$. It is therefore now the condition for having an increase in financial difficulties which is more severe than the condition $\alpha > \frac{1}{2}$ obtained for a symmetric increase in ambiguity. The reason is that, for a unilateral decrease in $\underline{\mu}$, we also have to add the effect of a decrease in μ_0, which, as underlined before, reduces financial difficulties.

Thus, when pessimism suficiently dominates, i.e. when we have $\alpha > 1 - \widetilde{\alpha}$, then any increase in ambiguity, whether symmetric or asymmetric, raises financial difficulties. At the opposite, when optimism sufficiently dominates, or equivalently when pesssimism is small enough, i.e when we have $\alpha < \widetilde{\alpha}$, then any increase in ambiguity reduces financial difficulties. In the intermediate case where we have $\widetilde{\alpha} < \alpha < 1 - \widetilde{\alpha}$, then the effect on ambiguity may depend on whether it takes more the form of an increase in $\overline{\mu}$ or of a decrease in $\underline{\mu}$, because in this case an increase in $\overline{\mu}$ raises financial difficulties while a decrease in $\underline{\mu}$ lowers them (with the magnitude of the corresponding effect also depending on the value of α in this interval).

4.3 Effect of pessimism or optimism

Consider the effect of the degree of pessimism α. From (31) we have

$$\frac{\partial g\left(\mu_j, \mu_0, \eta, \alpha\right)}{\partial \alpha} = 2\left[\eta + \psi(2\eta) - \frac{1}{2}\right] \tag{36}$$

As the distribution of $\varepsilon_j - \varepsilon_i$ is symmetric, we have $\psi(0) = \frac{1}{2}$. Consequently, when there is ambiguity ($\eta > 0$), we have $\psi(2\eta) > \frac{1}{2}$. This implies $\frac{\partial g(\mu_j, \mu_0, \eta, \alpha)}{\partial \alpha} > 0$. This means that an increase in the degree of pessimism raises financial difficulties. The reason is that more pessimism gives more weight to the worst case relatively to the best case. This raises financial difficulties through the two channels previously mentioned. First, for each player, the conditional weighted average of the expected values of the fundamentals θ is reduced because, in the worst case, it is the highest bias $\overline{\mu}$ (instead of the lowest bias $\underline{\mu}$ in the best case) which is substracted to the value of the signal received x_i. Second, as we have underlined before, in the worst case the degree of coordination expected by each player (the conditional expected value of l) is reduced (while it is increased in the best case). These two channels correspond to the coefficients 2η and $2\left(\psi(2\eta) - \frac{1}{2}\right)$ in (36), respectively.

These results can be summarized in the following Proposition:

Proposition 3 *In the first model (creditors financing a project):*

1. A higher value of the true bias μ_j reduces financial difficulties, while a higher value of the mean possible bias μ_0 increases financial difficulties. A simultaneous increase in each of the biases μ_j by some amount, with an equal increase in the mean possible bias μ_0, has no effect.

2. Effect of an increase in ambiguity:

- A symmetric increase in ambiguity, due to a greater value of η, increases financial difficulties in the case $\alpha > \frac{1}{2}$ where pessimism dominates, but decreases financial difficulties in the case $\alpha < \frac{1}{2}$ where optimism dominates.

- An asymmetric increase in ambiguity due to a greater value of the maximum possible bias $\overline{\mu}$ increases financial difficulties if and only if pessimism is not too small, i.e. in the case $\alpha > \widetilde{\alpha}$, where $\widetilde{\alpha}$ is given by (34) and satisfies $0 < \widetilde{\alpha} < \frac{1}{2}$.

- An asymmetric increase in ambiguity due to a smaller value of the minimum possible bias $\underline{\mu}$ increases financial difficulties if and only if pessimism sufficiently dominates (or equivalently if optimism is small enough), in the case $\alpha > 1 - \widetilde{\alpha}$.

3. If there is ambiguity, an increase in pessimism always increases financial difficulties.

5 Results for the second model: Comparison

From Proposition 1, if we want to know the effects on the equilibrium switching point k'^* and the implications in terms of financial difficulties, we simply

have to replace α by $1-\alpha'$, i.e. to replace pessimism by optimism, and vice versa[19]. Consequently, in the expression (28) we simply have to change the sign of coefficient $2\alpha-1$ in order to get the equilibrium switching point k'^*, which gives

$$k^{*\prime} = \frac{1}{2} + \mu_0' - (2\alpha' - 1)\left[\eta' + \psi'(2\eta') - \frac{1}{2}\right] \tag{37}$$

Let us compare the effects on financial difficulties in the two models, when we have the same parameters characterizing ambiguity and pessimism, i.e. when we have $\eta = \eta'$ and $\alpha = \alpha'$. To find the effects of the parameters, we therefore have to replace α by $1-\alpha$ in the analysis done with the first model.

In Section 4.1 above, the results which concern the effects of the true biases and of the average possible biais, in the first model, are unchanged when we replace α by $1-\alpha$. This implies that the effects are the same in second model, the currency attacks model with speculators.

The effects of more ambiguity were shown to depend on α. Therefore, we have to replace α by $1-\alpha$ in the conditions we found. When we consider a symmetric increase in ambiguity (an increase in η), this just gives opposite conditions: financial difficulties are increased when optimism dominates $\left(\alpha < \frac{1}{2}\right)$ and are reduced when pessimism dominates $\left(\alpha > \frac{1}{2}\right)$. As a consequence the effects are just opposite[20]. Thus, for example, in the case where pessimism dominates a symmetric increase in ambiguity increases financial difficulties in the first model, but decreases them in the second model. Therefore, the implications in terms of transparency would be opposite in the two models. In the case just considered where pessimism dominates, this would mean that liquidity crises would benefit from more transparency, but that exchange rate crises would be worsened by more transparency. The opposite holds in the case where optimism dominates.

When we consider the effects of an asymmetric increase in ambiguity , we have to add the effect of the implied value on the mean possible bias μ_0'. As a change in the mean possible bias has the same effects in the two models, and as a change in η has opposite effects in the two models, this implies that we will get opposite effects in the two models if the implied changes in the mean possible bias are opposite in the two models. Consequently, the effect

[19] Note that the best case and the worst case would have to be switched in the two models, and therefore Proposition 2 would have to be modified accordingly.

Also, as underlined in Section 2.3 above, the variable l' of the second model replaces the variable h of the first model, and the equilibrium switching strategy is for player i to play N' (and not C'), i.e. not to attack the currency, if $x_i' \geq k'^*$, and to play C' if $x_i' < k'^*$.

[20] This also clearly appears if we compare (28) and (37) , where the only change is that we replace $(2\alpha-1)$ by $-(2\alpha-1)$.

of an increase in $\overline{\mu}$ in the first model will be the opposite of the effect of a decrease in $\underline{\mu}'$ in the second model, and vice versa.

The effect of an increase in pessimism in one model is the same as an increase in optimism in the other model. Therefore, these effects are opposite in the two models. Thus, more pessimism increases financial difficulties in the model of creditors financing a project, but reduces them in the currency crisis model with speculators.

Thus, we get the following proposition:

Proposition 4 *If we compare the effects on the equilibrium switching point and on financial dificulties in the two models (when we have $\eta = \eta'$ and $\alpha = \alpha'$), we simply have to replace pessimism in one model by optimism, and vice versa (i.e. to replace α by $1 - \alpha$). Consequently, from Proposition 3, we get*

1. The effects of a change in the biases, or in the mean possible bias, are the same in the two models.

2. Concerning the effect of a change in ambiguity, we have:

- The effects of a change in symmetric ambiguity (a change in η) are opposite in the two models.

- The effect of an increase in $\overline{\mu}$ (or $\overline{\mu}'$) in one model is opposite to the effect of a decrease in $\underline{\mu}'$ (or $\underline{\mu}$) in the other model.

3. The effects of a change in pessimism α are opposite in the two models.

6 Conclusion

We have considered a simple linear global game model with a continuum of players and binary actions, where the signals received by the players may be biased, and where there is ambiguity on these biases. We have used a non-Bayesian approach to uncertainty where both optimism and pessimism can be taken into account. We have actually considered and compared two versions of this model which are inspired by two kinds of models of financial crises. The first model tries to represent a situation where each player is a creditor who has to decide whether to roll over the loan for some investment project or not. In the second model, each player is a speculator who has to decide whether to attack a currency or not. In both models, there are complementarities between some actions of the players (to roll over the loan, or to attack the currency, respectively). This creates the need for coordination between these actions. In each model, the action which needs coordination is also the action which has a payoff which depends on the level

of fundamentals. We have examined how the parameters of the model affect financial difficulties (as the model is linear we prefer to use the term "financial difficulties" rather than "financial crises")

We have shown that, in both types of model, the effect of ambiguity depends on the degree of pessimism versus optimism. When we consider a symmetric change in ambiguity, then the sign of its effect depends on whether optimism or pessimism dominates. This occurs through the two channels which have been underlined in the literature: through its effect on the expected value[21] of the fundamentals, on the one hand; and through its effect on the expected degree of coordination, on the other hand. In each model, a symmetric increase in ambiguity reduces the expected degree of coordination when pessimism dominates, but increases it when optimism dominates. (This actually means that, under ambiguity, the property of "Laplacian beliefs" underlined in the literature does not hold anymore: when the signal is equal to the switching point, each player does not anymore believe that the proportion of other players who take the action is uniformly distributed on $[0, 1]$). When pessimism dominates, this effect on expected coordination decreases (or increases when optimism dominates) the incentive that each player has to take the action which needs coordination. This effect going through expected coordination reinforces the effect going through the expected value of fundamentals.

If we consider asymmetric changes in ambiguity, due for example to an increase in the maximum possible bias, or to a decrease in the minimum possible bias, we find different conditions on the required level of pessimism. For, to the previous effect of a symmetric change in ambiguity, we have to add the implied effect on the mean possible bias. As a greater mean possible bias always increases financial difficulties, this gives a less stringent condition for having an increase in the maximum possible bias raise financial difficulties (and a more stringent condition for having a decrease in the mean possible bias increase financial difficulties) than in the case of a symmetric increase of ambiguity.

However, the effect of more ambiguity on financial difficulties depends on the model considered. In fact, we have shown that there is a symmetry between the two kinds of models which has its counterpart in the symmetry between optimism and pessimism: When we want to know the effect on the equilibrium switching point or on financial difficulties, we can go from the model of liquidity problems with creditors to the model of currency attacks with speculators by simply replacing pessimism by optimism, and vice versa.

[21] Here, by expected value we actually mean the weighted average of the expected values in the worst and best cases, as it appears in the criterion used by decision makers.

As a consequence, the effects of a symmetric change in ambiguity are exactly opposite in the two models. Thus, when pessimism dominates, more ambiguity increases financial difficulties in the model of liquidity problems with creditors, while it reduces fnancial difficulties in the currency attack model with speculators. In this case, more transparency would be beneficial in the first model, but would be harmful in the second model. The opposite is true when optimism dominates.

If we compare the effects of asymmetric changes in ambiguity in the two models, we find that an increase in the maximum possible bias in one model has an effect on financial difficulties which is opposite to the effect of a decrease in the minimum possible bias in the other model.

An increase in the degree of pessimism raises financial difficulties in the model of liquidity problems with creditors, but lowers financial difficulties in the model of currency atttacks with speculators.

We have also considered the effect of a shift in "market sentiment". Such a shift may correspond to a simultaneous increase or decrease in the biases of the signals of the players. In a way which has similarities with the results of Cheli and Della posta (2007), we find that when such a change in the bias is associated with an equal change in the mean posible bias held by the players, then nothing is really changed. But when this is not the case, and when, for example, the mean possible bias stays unchanged, then some real effect occurs. Thus, for example, if there is a rise in the biases and if there is no change in the mean possible bias held by the players, then financial difficulties are reduced. The opposite would occurs if there was an increase in the mean possible bias, with no change in the true biases. The crucial difference between these results and those of Cheli and Della posta (2007), who used an expected utility approach, is that such non neutral cases may be compatible with rationality in our framework of analysis, while it required some wrong expectations in their analysis. The reason is that having independent shifts either in the biases of the signals, or in the biases considered as possible by the players, is perfectly compatible with our non-Bayesian approach with ambiguity. The presence of ambiguity gives some room of maneuver for such independent shifts: as the players do not know the true biases, then, inside some interval, any values of the biases may be considered as possible.

As the signs of the effects of these shifts in market sentiment do not depend on the degree of pessimism, they are actually the same in the two kinds of models. Thus, a change in the degree of pessimim versus optimism has effects which are opposite in the two models, but a shift in market sentiment due to a change in the true biases, or a shift in market sentiment of the players due to a change in the average possible bias, on the contrary, have

similar effects in these two models. Therefore, when one talks about changes in the psychology, or mood, or sentiment of the participants in the financial markets, one has to be careful in interpreting these terms. For they can mean either a change in the behavior of players under ambiguity toward more pessimism or more optimism, or a change in the true biases, or a change in the mean posssible biases. And, as we have seen, the effects of these changes are not the same, and may or may not depend on the type of model.

According to our analysis, liquidity crises which occur because creditors do not roll over their loans should be very different, and in some sense opposite, to the currency crises due to speculators who attack a currency. They have opposite implications concerning the effect of ambiguity, the role of pessimism versus optimism, and the desirability of transparency. It would therefore be worthwhile that further research try to examine the empirical relevance of these findings.

Appendix

Condition on δ for having a non informative uniform prior on θ when we have $x_i = k^*$

From (20), the prior on θ is always non informative for the distribution of θ conditional on $x_i = k^*$, if, for any μ_i and any ε_i, then $k^* - \mu_i - \varepsilon_i$ always belongs to the support $\left[\frac{1}{2} - \delta, \frac{1}{2} + \delta\right]$ of the uniform prior on θ. This condition is satisfied if we have $k^* - \mu_i - \varepsilon_i \geq \frac{1}{2} - \delta$ and $k^* - \mu_i - \varepsilon_i \leq \frac{1}{2} + \delta$. As the support of ε_i is $[-\rho, \rho]$, and as μ_i belongs to $\left[\underline{\mu}, \overline{\mu}\right]$, these two inequalities are satisfied for all values of ε_i and μ_i if and only if we have $\delta \geq \frac{1}{2} - k^* + \overline{\mu} + \rho$ and $\delta \geq k^* - \frac{1}{2} - \underline{\mu} + \rho$. Using (19) and (28), these inequalities become $\delta \geq \eta + \rho - (2\alpha - 1)\left[\eta + \psi(2\eta) - \frac{1}{2}\right]$ and $\delta \geq \eta + \rho + (2\alpha - 1)\left[\eta + \psi(2\eta) - \frac{1}{2}\right]$. These two inequalities are equivalent to the inequality

$$\delta \geq \eta + \rho + (|2\alpha - 1|)\left[\eta + \psi(2\eta) - \frac{1}{2}\right] \tag{38}$$

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