Complementarity and the resource curse

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Abstract

This paper discusses how the economic structure and asset ownership shape economic and political outcomes. Using a simple model of the productive sector, I provide theoretical evidence that complementarities between productive assets reduce the stakes of political competition, and therefore reduce the intensity of the conflict over political power. In particular, these results provide a theoretical explanation for the frequent conflicts associated with abundant mineral resources. They are valid in a democratic setting, where this competition is electoral, but also in any other setting, where competition may be of a more violent nature. I then extend this analysis to show that complementarity of productive assets positively influences the willingness of elite groups to invest in property rights institutions, thus providing an economic explanation for why some countries have endogenously developed a context more favorable to business than others.

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1 Introduction

Recent advances in the political economy of development emphasize how crucial the quality of institutions is for economic development. In particular some suggest that the economic structure and asset ownership have a decisive role in shaping institutions and development outcomes (Acemoglu and Robinson, 2008; Besley and Persson, 2009, 2010; Bourguignon and Verdier, 2009, 2010). The celebrated case of the natural resource curse exemplifies such considerations. Resource-rich developing countries seem to be unable to successfully convert their exhaustible resource into long-term growth and asset accumulation (see van der Ploeg, 2011, for a recent survey). They also display more frequent and more violent conflicts (Collier and Hoeflller, 2004; Fearon, 2005), and poorer institutions (Bates, 2007). Several mechanisms explain how extractive activities can produce relatively worse economic outcomes, but why they should encourage rent-seeking behaviors, corruption, violent conflict over rent-appropriation and discourage investment in state capacity remains an open question. This paper offers a theoretical explanation for the conflictual nature of mineral extraction, and indeed extends the discussion to any other productive endeavor.

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Most economic activities involve two or more actors, characterized by their ownership (or rather here, control) of a productive asset. Here, I explore the consequences of asset ownership and of some characteristics of the production process on the likelihood and intensity of a conflict between the actors involved, and on their incentive to invest in state capacity. This allows me, as an illustration, to provide some new insights into the particularities behind two African success-stories. First, a very specific social context at independence accounts for the unequalled achievement of Botswana, which transformed its huge diamond wealth into a stable democracy and an impressive growth record (Harvey and Lewis, 1990; Tsie, 1995). Second, the textile industry and sugar cultivation transformed ethnic diversity in Mauritius into an opportunity and provided the ground for its peaceful, yet outstanding economic development, against very unfavorable odds at independence (Meade, 1961).

For that purpose, this paper provides a simple framework to examine the economic determinants of political competition. It therefore fits in recent strands of the literature, and it contributes in particular to the growing body of studies which emphasize the role of institutions in the development process (Hall and Jones, 1999; Acemoglu and Robinson, 2002; Rodrik et al., 2004; Acemoglu et al., 2005). Its focus is on institutions designed to protect property rights (Rodrik, 1999).

Specifically, individuals in a position to mobilize resources may find it profitable to invest in a common productive endeavor. This productive endeavor is described by a constant elasticity of substitution (CES) production function, in order to study the role of asset complementarity or, oppositely, substitutability in political and economic outcomes. The individuals have conflicting interests over the proceeds of this endeavor, even though they often find it profitable to involve the other in the production by offering him a share of the proceeds. Whose sharing preferences prevail is decided through a competitive political process. Political competition is modeled as a contest game between two such individuals willing to spend in order to have their own sharing preferences prevail. For each contestant, the stake of political power results from the different shares that he and the other would offer. Finally, actual implementation of the sharing rule depends on the quality of property rights institutions.

The model yields three main results. I first show that more substitutable

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1 Supermodular games may look as an alternative to this setting; they would however not lead to the results I am providing here. They offer a tractable framework for comparative statics, and indeed I use a variation of Topkis’s theorem in the proof of Prop. n°6, but here complementarity is one of the variables of interest. For a thorough review of supermodular games, see Vives (2005).

2 This is an unusual, if not unheard of, approach: most models of the influence of special interest groups over political competition are variations on classical electoral rules. For instance, Robinson et al. (2006); Acemoglu et al. (2004) consider an economic rent accruing to the group in power, which allows them to buy off their competitors or voters. Contest games focus more specifically on the costs and benefits of individuals who derive an economic advantage from political power. To my knowledge, prior to this paper, only Aslaksen and Torvik (2006) used a contest game to model the political competition in a rentier economy. In order to compare civil war with democracy, they claimed that a contest game could only account for an armed civil conflict. Their claim is contentious, however, as contest games can be argued to account for at least some important features of any form of the political selection process.

3 Technology is assumed to be independent from the political outcome, and both individuals have an identical objective function. The only organizational role and influence of the winning individual is setting the sharing rule.
assets result in higher stakes of political competition. The underlying intuition comes back to the definition of complementary assets: an asset is complementary with another if its investment increases the profitability of investing the other. The contest winner is willing to offer the other contestant a higher share of the proceeds of the common productive endeavor when the two assets are more complementary, in order to induce him to invest more. Sharing rules offered by both contestants would therefore be increasingly favorable to the other when complementarity increases, thus lowering the stakes of political competition.

The second result is the key result of the paper, and comes as a corollary of the first one: it establishes the link between the stakes of being in power and the intensity of political competition. Intuitively, the more the individuals have to gain from being in power, the more they are willing to spend in order to win the contest. It is reasonable to assume that the amount of wealth spent for the conquest of political power is a good proxy for the intensity of political conflict. As a consequence, my second theoretical prediction is that conflict over political power is likely to be more intense if the resources to be invested are substitutes, as in the case of the exploitation of mineral resources.

Once in power, the contest winner has a strong interest in ensuring the other’s participation in the common endeavor; but he may fail to convince the other that he would not renege on his promises. The third result establishes that greater complementarity of assets would be an incentive for this political leader to commit to uphold the sharing rule – indeed to uphold property rights. Despite its cost, the commitment mechanism (which can take the form of several formal institutions; classic ones are conditionalities imposed on Official Development Aid and an independent judiciary system) is therefore more likely to arise at times when and in places where assets are complements. Both the leader and the other actor benefit from the leader’s ability to commit.

Closest to this paper are Bourguignon and Verdier (2009, 2010). They study the effects of the economic structure of society, namely the complementarity of the productive resource controlled by the elite and those controlled by other social groups, on taxation, redistribution, and investment in state capacity. But while I share their initial intuition about the essential role of the economic structure of society, the present paper explores different political and institutional outcomes, thanks to a new framework. More generally, this paper builds upon the relatively recent literature in the field of political economy of development, led by Daron Acemoglu and co-authors. In particular, Acemoglu and Robinson (2006b) focus on the economic determinants of regime change, in particular from elite control to democracy, Acemoglu and Robinson (2006a, 2008) offer models of institutional persistence and change and the linkage from political to economic institutions. Acemoglu et al. (2002b); Acemoglu and Johnson (2005) convincingly claimed that previous population density and settler’s mortality have durably influenced the property rights institutions in former European colonies; Acemoglu et al. (2002a) go so far as using institutions to shed light on the idiosyncratic development narrative of one country, describing the customary legacy of independent Botswana as crucial in its stability and development.

Yet they overlook what may indeed have been key in avoiding the resource curse in Botswana.\textsuperscript{4} the interaction between individuals decisive in the fields

\textsuperscript{4}Botswana did not avoid the Dutch disease (Deléchat and Gaertner, 2008). But the growth of the diamonds sector compensated and concealed the forsworn industrial growth.
of both economics and politics (the economic structure of society). The issue of the effect of leaders on development outcomes was addressed only recently (Jones and Olken, 2005, 2009; Besley et al., 2011), and remains under investigation.\(^5\) However, even though it only appears implicitly throughout the paper, the assumption that the political contest opposes individuals with control over economic resources is crucial to my argument (Silve, 2012).

Another related strand of works is the growing body of literature on the determinants of state capacity. This literature focuses on one dimension of state capacity: fiscal capacity, defined as the ability to raise taxes (Besley and Persson, 2009; Bourguignon and Verdier, 2009; Cárdenas and Tuzemen, 2011). A notable exception is Besley and Persson (2010): they also consider a regulatory dimension of state capacity. They are in fine interested in accounting for property rights, but focus on a rather unorthodox definition of property rights enforcement: the regulatory capacity of the state is presented as the extent to which individuals can pledge their assets as collateral. In this paper, I will consider property rights enforcement in a perspective which is more frequent in the literature (Jones, 1981; De Long and Shleifer, 1993; Acemoglu et al., 2001), as protection from state predation. In a way, state capacity is here studied as the state’s ability to restrain itself.

The outline of the paper is as follows. In the next section I examine a simple productive framework where one individual sets the sharing rule of a common endeavor; I also provide a normative assessment of his decisions. In section 3, I consider a political contest over the privilege of setting the sharing rule. Therein are the two first theoretical results of the paper. In section 4, I introduce the possibility that the individual in power may renege on the promised sharing rule. Since this is anticipated by the other, he may find it profitable to invest in a costly commitment mechanism (third result). In section 5, I consider the history of two outliers of cross-country growth regressions, to illustrate how they fit my story. All proofs are relegated to the appendix.

2 The economic model

The economy, restricted to its productive sector, is composed of two decision-makers \(A\) and \(B\). They each control a different productive asset in quantities \(R_A\) and \(R_B\). Generically, \(A\) and \(B\) are individuals, and one’s asset is his own labor, but the model extends to groups who have solved the collective action problem, individuals in a position to mobilize others’ assets, and it also extends to other categories of productive assets, such as capital, skilled vs. unskilled labor etc. \(A\) and \(B\) are thus assumed to describe the economic structure of society. Assets can be used either in specific productive endeavors, described by technologies \(f_A\) and \(f_B\), or in a common productive endeavor, described by \(f\), which all produce an identical consumption good. Total production is therefore \(Y = f_A(x_A) + f_B(x_B) + f(x_A, x_B)\), where \((x_A, x_B) \in [0, R_A] \times [0, R_B]\) is used

\(^5\)This new literature assumes that certain leaders are “better” for development or at avoiding conflict. In the context of developing countries, political leaders are generally assumed to seek personal enrichment rather than ideally fostering widespread and inclusive growth. Benevolent leaders might however indeed be a part of the answer: political leaders both in Botswana and Mauritius were unanimously commended for avoiding conflict and fostering developmental policies. As an assumption, however, benevolence is unsatisfactory, as it teaches little to other developing countries, and has little predictive power.
for the common production while \((R_A - x_A, R_B - x_B)\) is used in the specific productive endeavors. This setup ensures that participation in the common production cannot be taken for granted and has to be made enticing; otherwise, they can invest in their specific activities, which will be assumed not to be taxable throughout the paper. In other words, the specific production function offer an outside option to both individuals. They are described by constant returns technologies: \(\forall k \in \{A, B\}, f_k(x_k) = \gamma_k(R_k - x_k)\). To study the effects of substitution or complementarity, the common productive endeavor is described by a CES function: \(f(x_A, x_B) = \gamma \left( \sum_k \mu_k x_k^{\sigma - 1} \right)^{\sigma / (\sigma - 1)}\). \(\mu_A, \mu_B = 1 - \mu_A\) are technological parameters; \(\gamma_A, \gamma_B\) and \(\gamma\) are efficiency parameters, and finally \(\sigma > 0\) is a substitution parameter between the two factors in the common production technology. Low values of \(\sigma\) correspond to more complementary assets; as \(\sigma\) grows, resources are more and more substitutable. For simplicity, I will thereafter only consider assets with an elasticity lower than 2, but the whole range is considered in the appendix.

Two parameters recurrently arise in the computations. First, \(\Gamma_i = \frac{\gamma_i^{\sigma - 1} \mu_i^\sigma}{\sum_k \mu_k R_k^{\sigma - 1}}\) can be interpreted as the profitability of investing \(i\)’s asset (implicitly: in the common endeavor) relative to using it for the specific production. For \(\sigma > 1\), \(i\)’s asset is said to be profitably invested in the common productive process rather than in \(i\)’s specific alternative when \(\Gamma_i\) is high. For \(\sigma < 1\), \(i\)’s asset is profitably invested when \(\Gamma_i\) is low. The complementarity between the two assets ensures that investment of one makes the common endeavor more profitable to the other. \(\Gamma_i\) characterizes only the intrinsic profitability of investing \(i\)’s asset, before taking into consideration investment of the other. Second, endowments are therefore usefully characterized by the parameter \(\beta_i = \frac{\mu_i R_i^{\sigma - 1}}{\sum_k \mu_k R_k^{\sigma - 1}}\); a high \(\beta_i\) in \([0, 1]\) indicates abundance of \(i\)’s asset relative to \(j\)’s, taking into account that the common production function may require more of one asset than of the other.

### 2.1 Feasible investments

The proceeds of the common endeavor are shared between both individuals, according to a sharing rule described by \((\alpha_A, \alpha_B)\), with \(\alpha_A + \alpha_B = 1\). The utility of individual \(i\) is given by \(U_i(x_A, x_B, \alpha_i) = f_i(x_i) + \alpha_i f(x_A, x_B)\). The problem verifies the necessary concavity conditions, and utility maximization provides me with the expressions of the two individuals’ best responses to each other’s investment:

\[
x_i^*(\alpha_i, x_j) = \min[R_i, \eta_i x_j] \quad \text{with} \quad \eta_i = \left( \frac{\mu_i R_i^{\sigma - 1}}{\sum_k \mu_k R_k^{\sigma - 1}} \right)^{\sigma / \sigma - 1}.
\]

(1)

For a given \(x_j\), \(\eta_i\) increases with the relative profitability \(\Gamma_i\) of the common endeavor and with \(\alpha_i\). It decreases with \(\sigma\). \((0, 0)\) is the unique and stable Nash
equilibrium iff $\eta_A \eta_B < 1$. Graphically, this provides me with four situations\(^9\)
(cf. Fig n°1), with one stable Nash equilibrium $(x_A^*(\alpha), x_B^*(\alpha))$ each. Upon
choosing $\alpha$, a social planner may face a choice between implementing several
of these equilibria: for instance, a small $\alpha_A$ (and thus big $\alpha_B$) may lead to
full investment of $B$’s asset but partial of $A$’s, while a big $\alpha_A$ may produce the
opposite outcome. A range of $\alpha$s might all implement full investment of both
assets. I define feasibility of a situation as existence of an $\alpha$ leading to investment
decisions by both individuals generating that situation in equilibrium.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Four possible Nash equilibria resulting from different values of the parameters.}
\end{figure}

**Lemma 1.** Feasibility of the various investment equilibria is usefully described
by four ranges of parameters:

- if both assets are profitably invested in the common endeavor, full invest-
  ment of both assets is feasible. In this range, full investment of either asset
  and partial investment of the other is always feasible;

- if neither asset is profitably invested, no sharing rule can lead to any in-
  vestment in the common endeavor;

\(^8\)Considering $\eta_A$ and $\eta_B$ to be functions of $\alpha_A$, the equation $\eta_A \eta_B = 1$
can be shown to have 0, 1 or 2 solutions in $[0, 1]$. When it has two solutions, I will refer to them as $\alpha_1$ and $\alpha_2$, with $\alpha_1 < \alpha_2$. When it has only one solution, I will refer to it as $\alpha_1$ or $\alpha_2$ by continuity. The motivated reader will find details in the appendix.

\(^9\)plus the two situations arising from $\alpha_i \in \{0, 1\}$, which can be defined by continuation and have obvious investment consequences.
• the two previous cases define adjacent ranges, and unambiguously define two complementary ranges. In each of these ranges, one asset is profitably invested while the other is not. In the corresponding range, only full investment of the former asset and partial of the latter is feasible.

![Diagram](image)

Figure 2: Feasibility of the various non-(0,0) situations (left: \( \sigma < 1 \), right: \( \sigma \in [1, 2] \)); shaded, no nonnull equilibrium

Lemma n°1 characterizes the domain of implementation of each equilibrium. It is illustrated by Fig. n°2. The first range can be characterized by the full investment threshold: \( \sum_k \frac{\beta_k}{1 - \Gamma_k} \leq 1 \), and the second by the no investment threshold: \( \sum_k \frac{\Gamma_k}{1 - \sigma} \geq 1 \). The two thresholds are tangent for \( \frac{\beta_i}{1 - \Gamma_i} = \sum_k \frac{\beta_k}{1 - \Gamma_k} \). This lemma provides interesting insights. Rather obviously, i’s asset is more readily fully invested when i finds the common endeavor profitable (for \( \sigma > 1 \), \( \Gamma_i \) high enough, and the converse for \( \sigma < 1 \)). Investment of one asset makes investment of the other more profitable. This positive spillover is higher if the two assets are more complementary. As a consequence, as complementarity increases, the share that each individual demands in order to fully invest decreases, and the range of parameters for which both assets are fully invested increases. Conversely, the more the two assets are substitutable, the higher the share one individual demands in order to fully invest.

One last observation deserves a mention: for any \( \sigma < 2 \), there is a range of parameters \( \left( \sum_k \frac{\beta_k}{1 - \Gamma_k} \leq 1 \right) \) in which neither asset, if alone, would be invested in the common productive endeavor, but in which the two assets can be profitably invested together (this is not true anymore for \( \sigma \geq 2 \)).

### 2.2 Optimal sharing rule

Assume it is one of the two individuals’ privilege – A’s for instance – to set the sharing rule. He faces a trade-off between enticing B with a fair share of the

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10This is not true anymore for \( \sigma > 2 \), thus giving rise to slightly more complex considerations. All cases are considered in the appendix.
common endeavor and keeping as much for himself as possible. Formally, his program can be written:

$$\max_{\alpha_A, \alpha_B, x_A} U_A(x_A, x_B, \alpha_A)$$

s.t.  
\[
\begin{align*}
\alpha_A &\in [0, 1] \\
\alpha_B &= 1 - \alpha_A \\
x_B &= x_B(\alpha_B, x_A)
\end{align*}
\]

(2)

The previous analysis remains valid. Therefore A’s program can be simplified to setting $\alpha$ so as to maximize his indirect utility $V_A(\alpha) = f_A(x_A^*(\alpha)) + \alpha_A f(x_A^*(\alpha), x_B^*(\alpha))$: it is a nonconstant continuous function, and therefore has a maximum in $[0, 1]$, with maximand $\alpha^* = (\alpha_A^*, \alpha_B^*)$.

**Prop. 1.** A’s optimal sharing rule is usefully described by four ranges of parameters:

- if both assets are profitably invested, A offers $\alpha_B = \frac{\beta_B}{1 - \alpha_1}$, which results in full investment of both assets;
- if B’s asset only is profitably invested, A still invests part of his own asset, and offers $\alpha_B = 1 - \alpha_2$ so as to induce full investment of B’s;
- if A’s asset only is profitably invested, he still finds it profitable to offer B to participate. He offers $\alpha_B = 1 - \alpha_1$ for $\Gamma_B$ over a threshold, and $\alpha_B < 1 - \alpha_1$ under it. He fully invests his own asset;
- if neither asset is profitably invested, setting a sharing rule is pointless.

![Figure 3: Equilibria resulting from A’s optimal choice of sharing rule (left: $\sigma < 1$, right: $\sigma \in [1, 2]$).](image)

Prop. n°1 characterizes A’s choice of sharing rule for any value of the parameters. It is illustrated by Fig. n°3. When B’s asset is profitably invested (for $\sigma > 1$, $\Gamma_B$ high enough, and for $\sigma < 1$, $\Gamma_B$ low enough), A benefits from having B fully invest his asset, and he maximizes his own share conditional on inducing full investment from B. When B’s asset is not so profitably invested, setting a sharing rule only makes sense if A’s asset is profitably invested. Assuming
it is, A always sets the sharing rule so as to maintain his own full investment. Among possible such sharing rules, he does not maximize his own share, as he benefits from B’s investment. If the complementarity between their assets is high enough (σ ≤ 1), in fact, in the set of such sharing rules, he offers B the highest share available. The complementarity effect is more efficient from his point of view than the appropriation effect. If the complementarity is intermediate (σ ∈ [1, 2]), he only offers B the highest share available if Γ_B is not too low. If Γ_B is too low, A may find maximizing B’s share too costly at some point. Instead, he offers him an intermediate share. A corollary to Prop. n°1 is that investment of both assets, as a result of A’s choice of sharing rule, is nondecreasing in σ.

2.3 Normative benchmarks

How does A’s offer compare to a hypothetical social planner’s program? A social planner with full control of asset allocation would simultaneously choose x_A and x_B to maximize Y. Formally, her program can be written:

\[
\max_{x_A, x_B} Y(x_A, x_B) \\
\text{s.t. } \begin{cases} 
  x_A \in [0, R_A] \\
  x_B \in [0, R_B]
\end{cases}
\]  

Prop. 2. “First-best efficient” asset allocation
The social planner’s investment decision can be characterized by four ranges of parameters:

- she invests nothing (x_A = x_B = 0) if the two assets are jointly not profitably invested;
- she invests fully both assets (x_A = R_A, x_B = R_B) if they both are sufficiently profitably invested;
- otherwise, one asset i is profitably invested. She invests this asset fully (x_i = R_i), and the other partially (x_j = (\mu_i / (1 - \mu_j))^{-\gamma_j / \sigma} R_j).

Prop. n°2 characterizes the Y-maximizing program of a unique individual in control of both assets. It is illustrated by Fig. n°4. This individual’s investment decisions reflected the investment decisions which followed from A’s optimal sharing rule decision (as described by Prop. n°1) when and only when σ = 1. The no-investment threshold is described by the equation \( \sum_k \Gamma_k = 1; \) and the full-investment threshold by \( \min_k \Gamma_k = \beta_k. \) Even when σ = 1, however, A’s program also entailed distributional consequences which are meaningless here. In particular, notice that when both assets are profitably invested, A and...
Figure 4: First-best efficient asset allocation (left: \( \sigma < 1 \), right: \( \sigma > 1 \)).

\( B \) would each set a different sharing rule (to reach the same investment decisions). \textit{A contrario}, when \( \sigma \neq 1 \), investment decisions based on an individual’s program are inefficient (in terms of production maximization) for several ranges of parameters. There are two separate factors which generate this inefficiency: first, as there are no transfers here, the Coase theorem does not apply here, and \( A \)’s interest may sometimes not coincide with production maximizing. Second, even in the perspective of production maximization, when investment decisions are decentralized, each agent does not value the positive spillovers his investment generates on the other’s. To study the latter factor, let me examine the program of a social planner who would control the sharing rule, and not directly asset allocation:

\[
\begin{align*}
\max_{\alpha} & \quad Y(x_A, x_B) \\
\text{s.t.} & \quad \alpha_A \in [0, 1] \\
& \quad \alpha_B = 1 - \alpha_A \\
& \quad x_A = x_A^{**}(\alpha) \\
& \quad x_B = x_B^{**}(\alpha)
\end{align*}
\]

Prop. 3. “Second-best constrained” asset allocation

The social planner’s choice of sharing rule can be characterized by four ranges of parameters:

- whenever full investment is feasible, that is when both assets are jointly profitably invested, she sets any \( \alpha_A \in [\frac{\beta_A}{R_A}, 1 - \frac{\beta_B}{R_B}] \) to induce full investment of both;

- if no investment is feasible, that is if neither asset is profitably invested, setting the sharing rule is pointless (\( x_A = x_B = 0 \) anyway);

- otherwise one asset is profitably invested. If it is \( A \)’s, she sets \( \alpha_A = \alpha_1 \) so as to maximize investment of \( B \)’s asset, while ensuring full investment of \( A \)’s. If, conversely, it is \( B \)’s, she sets \( \alpha = \alpha_2 \).
Figure 5: Second-best asset allocation with property rights (left: $\sigma < 1$, right: $\sigma \in [1, 2]$)

Prop. n°3 characterizes the $Y$-maximizing program of a social planner hypothetically in charge of setting the sharing rule. It is illustrated by Fig n°5. Comparison with Prop. n°4 illustrates how the introduction of property rights, in other words how decentralizing investment decisions, constrained the possible investment outcomes. It obviously captures most of the inefficiency previously discussed. Only when $\sigma = 1$ is the social planner in a position to implement the first-best asset allocation for any values of the parameters. For $\sigma \neq 1$, she will only be able to reach the first-best investments for limited ranges of parameters, that is under the first-best no-investment threshold (no investment) and over the second-best, or decentralized, full-investment threshold (full investment). Both too much and not enough complementarity between the two assets may be a source of inefficiency, and constrain the feasible investments.

Comparison of Fig. n°5 with Fig. n°1 illustrates that A’s program entails one additional source of inefficiency relative to the program of a production-maximizing social planner: he is sometimes in a position to increase his utility at the cost of some overall production. For $\sigma > 1$ and $\Gamma_B$ low enough (B’s asset is not so profitably invested), A would offer B a lower share than the social planner would.\footnote{In fact, for $\sigma > 2$, there is even a range of parameters in which A would keep the whole proceeds of the common endeavor for himself ($\alpha_A = 1$), thus effectively excluding B from the formal sector. The cost of attracting A’s investment would be higher than the gains.} B’s investment is therefore suboptimal from the social planner’s point of view.

Finally, when a range of $\alpha$s generate the same investment strategies (in this instance, this occurs only for the full-investment case), while the social planner may be indifferent over redistributive issues, A obviously is not, and maximizes his own share.
3 The political model

3.1 The stakes of political power

For the reasons just outlined (low profitability of the investment of one asset, redistributive issues), A and B often do not set the same sharing rules. While for a given state of the world A would set \( \alpha^A = (\alpha^A_A, \alpha^A_B) \), B favors \( \alpha^B = (\alpha^B_A, \alpha^B_B) \) and would set it if in power. Notice that B’s program is exactly symmetrical to A’s in the \((\Gamma_A, \Gamma_B)\)-plane. The indirect utilities resulting from such sharing rules are \( V_A(\alpha^A_A) \) and \( V_B(\alpha^A_B) \) on the one hand, \( V_A(\alpha^B_A) \) and \( V_B(\alpha^B_B) \) on the other. Let me define \( v_A = V_A(\alpha^A_A) - V_A(\alpha^B_A) \) and \( v_B = V_B(\alpha^B_B) - V_B(\alpha^B_A) \): since \( \alpha^i \) maximizes \( V_i \), \( v_i \) is necessarily nonnegative. These scalars fully capture the economic value for A and B of obtaining control over the sharing rule, the stakes of competition for political power.

Prop. 4. Complementarity and propensity for conflict: The stakes of political competition are higher for more substitutable assets. In other words, \( v_A \) and \( v_B \) are nondecreasing in \( \sigma \):

\[ \forall k \in \{A, B\}, \frac{dv_k}{d\sigma} \geq 0. \]

Prop. n°4 is key in addressing the question raised in this paper: why different activities, even if they are similarly efficient, can have vastly different political economic outcomes. Mineral extraction, in particular, is associated with high returns and low complementarity between individuals in a position to manage it. As a consequence, there is little incentive to redistribute widely its proceeds, which raises the stakes of controlling the extractive process. As already mentioned in the introduction, Prop. n°4 is intuitive once the true nature of the complementary of substitutable assets has been grasped. If A is in power, he will find it more profitable to involve B in the common productive endeavor if their assets are complementary rather than substitutes, and as a consequence, he is likely to offer him a higher share of its proceeds. If B is in power, the reasoning is similar. As a consequence, when \( \sigma \) decreases, one expects the gap between the two alternative sharing rules to diminish, thus reducing the stakes of political power. For all the simplicity of the idea, the proof is rather tedious, but it can be found in the appendix.

Finally, the cross-derivatives of \( \frac{d^2v_A}{d\sigma d\gamma} \) and \( \frac{d^2v_B}{d\sigma d\gamma} \) are positive. Profitability and substitutability are both at the heart of the possible conflict between A and B, and they tend to reinforce one another as factors of discord.

3.2 Political competition

So far, the model does not include a mechanism to select who gets to decide. I model political competition as a contest game. Each contestant \( k \) provides an outlay \( b_k \) in order to win a prize. The prize is here the utility gain he would derive from setting the sharing rule, rather than the other setting it. The probability of him winning increases with his own outlay and decreases with

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13In fact, that A and B sometimes DO set the same sharing rules is only an artifact of this model. It is a consequence of the frequency of corner solutions, a necessity if the model is to be solved, but it should not be further interpreted.
the other’s. Outlays are sunk costs, which can be considered as pledges on the future proceeds of the common productive endeavor on financial markets (which are not explicitly modeled).

An extensive literature has identified numerous contest success functions (the technology that translates the individual efforts into probabilities of winning the contest). The most common are the perfect discrimination (the highest outlay wins) and the Tullock contest.\textsuperscript{14} Both settings offer the same insight: Tullock (1980); Hillman and Riley (1989) have previously shown that equilibrium outlays \((b^*_A, b^*_B)\) increase with the value of the prize, indeed that individual outlays increase as each individual values the prize more.\textsuperscript{15} Thanks to Prop. n°4, the second result immediately follows:

**Prop. 5. Complementarity and actual conflict:**

The intensity of the conflict for political power decreases with the complementarity of the assets. In other words, \(b_A\) and \(b_B\) are nondecreasing in \(\sigma\):

\[
\forall k \in \{A, B\}, \frac{db^*_k}{d\sigma} \geq 0.
\]

Even though the gains from setting the sharing rule, \(v_A\) and \(v_B\), cannot formally be labeled as a political rent, they have the same intuitive characteristics: they are benefits from being in power which are not linked to any activity beneficial to the economy as a whole. On the contrary, they are extracted at the cost of reaching a suboptimal level of production. It is therefore useful to think of them as a rent derived from political power. Another way to formulate Prop. n°5, therefore, is that higher substitutability results into more rent dissipation. While Prop. n°4 made clear that the political rent decreased with the complementarity of the assets, Prop. n°5 concludes that indeed conflictuality (seen here as the amount of effort spent in the political contest) decreases with the complementarity of assets.

The cross-derivatives \(\frac{d^2b^*_A}{d\sigma d\gamma}\) and \(\frac{d^2b^*_B}{d\sigma d\gamma}\) are positive: as before, profitability and substitutability tend to reinforce one another as factors of conflicts. Let me also mention that the contest technology and the degree of asymmetry between the stakes of power may also affect the intensity of the conflict. This is however not central to this analysis.

### 4 Property rights institutions

I have so far assumed that once set, any sharing rule is enforced with certainty. The individual in power never reneged on his offer once production had taken place. In other words, I have implicitly assumed perfect protection from the predation of the state. Meanwhile, there is a growing consensus among economists, that expropriation by the government or powerful elites is a decisive hindrance to development. Checks against state predation – in other words, institutions

---

\textsuperscript{14} Player \(i\) provides an outlay \(b_i\) with probability of winning \(\frac{b^*_i}{b^*_A + b^*_B}\). Parameter \(r > 0\) characterizes the contest technology, with increasing returns (high \(r\)) or decreasing returns (low \(r\)).

\textsuperscript{15} They provide explicit expressions of the outlays. Those depend on the contest technology, on the asymmetry between the stakes for each player. Moreover, given the asymmetry and the technology, outlays are proportional to one’s stake in the contest.
of property rights protection—have indeed been shown to be good predictors of long run economic development (Acemoglu et al., 2001, 2002b; Acemoglu and Johnson, 2005).\footnote{This consensus reaches beyond the academic community. The usual governance or investment climate indicators provided by international financial institutions all rely on state predation in their definitions.}

While the previous section examined the political features of a simple productive framework, this one relaxes the assumption of perfect property rights or, equivalently, of effective enforcement of the sharing rule. These two sections are independent. From now on, one individual is assumed to be in power; how he reached that position is of no consequence.\footnote{In particular, I do not examine how the prospect of having to invest in property rights institutions may affect the stakes of winning the contest game introduced in the previous section.}

Assume \( A \) is in power. He cannot be counted upon to meet his commitment once production has taken place. In other words, there is no property rights protection. \( B \) expects that whatever \( A \)'s offer he will in fact get nothing. Without credible enforcement of the sharing rule, there can therefore be no common production.

Assume now \( B \) expects \( A \) to maybe renege on the initial offer (partial property rights protection): with probability \( p \), he gets only his own specific production, and with probability \( 1 - p \) the sharing rule is effectively enforced. This situation corresponds for instance to an uncertain judicial or political environment: \( A \) may be able to corrupt the judge or the politician (but he cannot count upon successfully doing so), or maybe the enforcement scheme is itself found lacking. Such a scheme can also rely on an exterior third actor, such as in the case of conditional development aid: in that case reneging on the contract may provoke the interruption of financial disbursements and thus entail immediately adverse consequences in terms of \( A \)'s utility. Anyway, a high \( p \) means poor property rights enforcement, and a low \( p \) good property rights. If \( A \) offers the sharing rule \( \alpha^{Ar} = (\alpha^{Ar}_A, \alpha^{Ar}_B) \) (\( r \) standing for ‘reneging’), \( B \) gets an expected share of \((1 - p)\alpha^{Ar}_B\), and \( A \) \((1 - p)\alpha^{Ar}_A + p\).

Let me now assume that it is possible to invest in the commitment capacity of the state. Formally, the state commits to restrain itself from predation. To increase the credibility of the professed sharing rule, \( A \) reduces the likelihood of himself reneging by investing in an external commitment mechanism. To reach a level \( p \), he pays a cost \( C(p) \), nondecreasing: a higher commitment involves higher costs. This typically corresponds to the cost of an independent judicial system, but one could also imagine other schemes: for instance, conditional development aid associates a cost with a breach of the property rights, which is likely to reduce the incentive for the state to renege.\footnote{Conditional aid would be best accounted for, however, by a liability contingent upon reneging. The results would not substantially differ.} Formally, his full program can thus be written:

\[
\max_{\alpha_A, \alpha_B, x_A, x_B, p} U_A(x_A, x_B, \alpha^{Ar}_A) - C(p) \quad \text{s.t.} \quad \begin{align*}
\alpha_B & = 1 - \alpha_A \\
p & \in [0, 1] \\
x_B & = x_B^* ((1 - p)\alpha_B, x_A)
\end{align*} \tag{5}
\]
A would set $\sigma^{Ar}$ such that it maximizes $V_A((1-p)\alpha + p)$. Let me write $\sigma^A$ the sharing rule he would have implemented under perfect property rights protection (as derived from Prop. n°1). Whenever possible, he would set $\sigma^{Ar} = (\frac{\sigma^A + p}{1 - p}, \frac{\sigma^A}{1 - p})$. With no constraint on the sharing rules, partial property rights protection entails uncertainty on the utilities of the players, but $A$’s could still fully compensate $B$. He expectedly reaches the same outcome as with perfect property rights protection: $V_A^*(p) = V_i(\alpha_i^*)$ (constant in $p$). $A$’s final valuation can be written $W_A(p) = V_A^*(p) - C(p)$.

The model, however, rests on the assumption that the specific productions of each individual cannot be taxed or otherwise captured. In other words, the liability of $A$ and $B$ is limited to their participation in the common endeavor. No sharing rule outside $[0, 1]$ can be offered credibly. $A$’s program can thus be simplified into:

$$\max_{p \in [0, 1]} W_A(p)$$  \hspace{1cm} (6)

Suppose $\sigma^A \geq p$, then $A$ can offer $B$ $\alpha^{Ar} \in [0, 1]$: he is able to fully compensate him for his own inability to commit.\(^{19}\) Now suppose $\sigma^A < p$, $i$ cannot offer $\alpha^{Ar} > 1$. He can possibly offer $\alpha^{Ar} = 1$: he retains ownership of the common endeavor if he successfully renews, with probability $p$, and relinquishes everything if he does not, with probability $1 - p$. Possibly if $p$ is high enough the resulting investment leaves $A$ worse off than if he retained the full common production, even at the cost of $B$’s participation. Let me define $\overline{p}$ the threshold above which $V_A^*(\alpha^{Ar}) = V_A(\alpha^A = 0) = V_A(\alpha^A = 0)$. Notice that $\overline{p}$ is possibly $1$.

$V_A^*$, considered as a function of $p$, is nonincreasing, constant equal to $V_A(\alpha^A)$ for $p \leq \alpha^A$, constant equal to $V_A(1)$ for $p \geq \overline{p}$, and decreasing in between. It is continuous everywhere. As expected, a higher $p$ has therefore an increasingly adverse impact on $A$’s utility. Fig. n°6 illustrates the sharing rule resulting from $A$’s inability to commit, and the utility he derives from it. The set of the Nash investment equilibria feasible for a given set of parameters is a subset of those which were previously derived with perfect property rights protection: some equilibria, however, may not be feasible any more.

$W_A$ is increasing in $p$ over $[0, \alpha^A]$ and over $[\overline{p}, 1]$. It is not necessarily monotonic in between. Once in power, $A$ sets $p^*_A \in [\alpha^A, \overline{p}] \cup \{1\}$ as the maximand of $W_A$. It is possible that he has no incentive to invest in such a scheme (resulting in $p^*_A = 1$); but it is not hard to see that any choice outside that set would be dominated by another within it. $p^*_A$ can be interpreted as the optimal static level of property rights protection from $A$’s point of view, once the cost of investing in the commitment mechanism is taken into account. My third and last result focuses on this optimal level of property rights protection:

**Prop. 6. Contract-enforcing institutions and complementarity**

The political leader’s investment in property rights institutions increases with the complementarity of assets. In other words $p^*_A$ and $p^*_B$ are nondecreasing in $\sigma$:

$$\forall k \in \{A, B\}, \frac{dp^*_k}{d\sigma} \geq 0.$$  

\(^{19}\)Agents are risk neutral; risk-averse agents would request a risk premium as a compensation for the insecurity of the sharing rule, which would only reinforce the mechanism under scrutiny.
Figure 6: A’s optimal offer with imperfect commitment (left) and his resulting utility (right), as functions of $p$.

An individual in power has a higher incentive to commit (lower $p^*_i$) to upholding his sharing rule offer for more complementary assets (lower $\sigma$), and is therefore more willing to invest in a costly $p$-reducing scheme. He expects to profit from the additional investment it would induce from the other. Notice that the mechanism which is highlighted here is not only very intuitive, it is also static (the proof makes an argument similar to that of Topkis’s theorem). The dynamic dimension of investment in state capacity brings additional insights which I do not address here, such as the link between investment in state capacity and the risk of losing power (Bourguignon and Verdier, 2009, 2010; Besley and Persson, 2010); but a static framework is enough to provide interesting insights.

5 Country case studies

The previous theoretical results offer new insights into the stylized facts mentioned in the introduction: mineral resources tend to be associated with more conflict (one major factor behind the resource curse) and with less state capacity. A secondary puzzle of Barro (1991)’s cross-country growth regressions is the astonishing development of some countries, such as Botswana and Mauritius which, based on their demographical and geographical characteristics, on their resource endowments, and on average development outcomes in comparable countries, should have remained poor and marred by civil conflicts. Not only did they achieve very high and sustained levels of growth since their independence, but all indicators of governance, competitiveness, and democracy position them in the top spots in Africa. To the day, there has been very few successful attempts to explain the peculiarity of these success-stories (Silve, 2012, provides a review of the literature on both countries, and offers a more detailed assessment of their success); the three theoretical results of this paper shed some light on some factors that underline their seemingly odd development trajectories.
5.1 Mauritius

Mauritius is a densely populated, fertile island in the Indian Ocean, with no known mineral resources. At independence, in 1968, its economy was among the poorest of the world, with a central role of sugar cultivation, which represented as much as 20% of its GDP and 60% of export receipts. Commissioned by the government of Mauritius to study the development prospects of the country, Meade (1961) pointed that ethnic heterogeneity would prove the most important obstacle to growth, and indeed, even though recent studies have nuanced this view, this intuition has been largely confirmed (Alesina and La Ferrara, 2005).

Sugar cultivation and textile production were the two very profitable sectors instrumental in initiating forty years of very dynamic average growth in Mauritius. These productive sectors / common endeavors were unskilled labor-intensive, and thus displayed a high complementarity between different groups of workers. Mauritius managed to grow fast for a long time and to avoid the social conflicts that some believed would be inevitable in such an ethnically diverse polity. Other countries with similar endowments are not particularly prone to conflicts either; Mauritius managed to grow faster than them, though, thanks to the exceptionally favorable terms it was granted in the sugar sector by the Lomé convention, and in the textile sector by the Multi Fibre Agreement. The following steps of the Mauritian development are well-known: tourism, export-processing zone, offshore business and finally an outsourcing industry were all characterized by a high complementarity between workers. In this context, ethnic diversity had no reason to be a hindrance to political stability and economic prosperity. Prop. n°4 predicts the political conflict entails low stakes, Prop. n°5 that the conflict was unlikely to become violent, and Prop. n°6 that complementarity would be an incentive to invest in the regulatory capacity of the state. How do those predictions compare to the facts?

First, growth in Mauritius has remained very inclusive over the years, and as a result inequalities are low from an international perspective. Second, according to the Economic Intelligence Unit, a London-based company within The Economist Group which publishes an index of democracy around the world, Mauritius is ranked first in Africa for the quality of its democratic process, on par with most OECD countries. Power was already peacefully transferred several times between competing political parties since independence, a unique achievement among countries in Africa and in the Indian Ocean. Third, prominent indicators of institutional quality (Doing Business and World Governance Indicators of the World Bank, Global Competitiveness indicator of the World Economic Forum and the Ibrahim index of governance) all position Mauritius within the first three spots in Africa. Overall, this country is a perfect illustration of how complementary assets tend to generate a peaceful development and good regulatory institutions.

5.2 Botswana

Botswana is a landlocked country in Southern Africa, largely covered by the Kalahari desert, with abundant diamond resources discovered after independence (in 1966). Diamonds in Botswana were found in kimberlite pipes, in other words in mines (Orapa, Jwaneng, Lethakane, Damtchaa), very early shown to be among the most important in the world. Point-source resources are often
quoted as the surest predictor of the resource-curse (for a recent review of the resource curse literature, see van der Ploeg, 2011); relative to its neighbors with alluvial diamonds, such as DRC, all factors pointed toward violent conflict. Yet Botswana has been able to successfully transform its huge diamond wealth into a stable democracy and an impressive growth record over the past forty years, a unique achievement among diamond-rich countries.

That is not to say that there was no potential for conflict in Botswana. Indeed diamonds did raise the stakes of political competition, as Prop. n°4 predicts. Inequalities there are among the highest in the world, and the ethnic minority remains extremely poor to the day\footnote{Income inequalities are the only measurable inequalities. This picture would ideally need to be nuanced to take into account non-income redistribution, for instance through a reputable health care system.} (the population of Botswana is small and relatively homogeneous: 79% describe themselves as belonging to the Tswana ethnic group). Moreover, democracy in Botswana, even though never caught out, is often described as “flawed”, as it has never been tested by a majority change. Why outright conflict did not erupt in Botswana, however, remains apparently paradoxical at this stage.

What differentiated Botswana from its neighbors stems from the economic structure of the society at the time of independence. Seretse Khama had been the heir to the customary chiefship, wealthiest and most powerful among the leading class of cattle-owners in Bechuanaland; he had renounced his customary rights in order to negotiate Botswana’s independence from the British colonial power, and he had been elected with an overwhelming majority president of the newly independent Botswana. Finally, the lands where diamonds had been discovered were under his authority, yet he had nationalized them.\footnote{Khama’s legitimacy was further reinforced by several factors which are best described in Acemoglu et al. (2002a).} In no other country were so much evidence gathered to support the rights of one group to manage the extraction of one resource and indeed, the political party he had formed remains in power to this day in Botswana. One essential assumption of Prop. n°5 is that two individuals who compete in the political arena are also those in a position to mobilize resources (or to solve the collective action problem). This assumption is not very far-fetched, as political competition is here modeled as having only economic stakes; but it is far-reaching. In newly independent Botswana, as in Norway or the US when they discovered oil, there was virtually no economic or political actor in a position to dispute the legitimacy of the state, neither internal nor external. This is quite obvious in the case of more advanced nations, and the evidence gathered shows that it was also the case in Botswana. Conflict remained exclusively democratic, and never became violent, thanks to the absence of any contender in a position to mobilize resources for conflict. In other countries, where a contender could arise, the result was systematically violent civil conflict.

Lastly, Prop. n°6 is not interestingly illustrated by the history of Botswana. The origins of the development of the state’s regulatory capacity (previously mentioned indicators of institutional quality all position Botswana just behind Mauritius, in the top spots in Africa) should rather be attributed to a dynamic view of state capacity building. The unlikelihood of losing power was certainly decisive in the decision by the authorities to invest in a regulatory framework which often tops African rankings (cf. Bourguignon and Verdier, 2009, 2010;
6 Conclusion

Mineral extraction is indeed, in this paper’s perspective, a common productive endeavor characterized by high substitutability and, in times of high market prices, high profitability. I have provided a mechanism whereby abundant mineral resources tend to be associated with more frequent and more violent conflicts, and a new intuition for why mineral-rich countries did not develop good property rights institutions and, more generally, regulatory state capacity. Political competition is essentially motivated by the gain of sharing the proceeds of the common production according to one’s own interests, a gain which is lower when the productive assets are more complementary. Since conflicts are costly, and even possibly destructive, it is fair to assume that this mechanism offers one more explanation to the resource curse.

This mechanism extends to encompass any kind of activity, which it characterizes by the level of complementarity it implies between productive resources. The history of Mauritius illustrated how labor-intensive activities, with high complementarity within initially abundant unskilled labor, promote inclusive growth, a peaceful democratic process and good property rights institutions. On the contrary, diamond extraction in Botswana did raise the stakes of political competition, which did not develop into civil conflict thanks to a very centralized polity. One implicit assumption, that of the economic structure of society, has been key throughout that paper, and should be further explored: individuals who compete in the political arena are also those in a position to mobilize resources. It allowed me to differentiate diamond-rich Botswana from countries with similar endowments, and it explained why, even though ethnic division had been thought to be a very likely source of conflict and a major impediment to growth in Mauritius, a peaceful polity naturally arose.

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References


Appendix

A  Feasible investments (proof of Lemma n°1)

If there exists an $\alpha$ such that $\eta_A\eta_B \geq 1$, then investment in the common endeavor by both asset holders is feasible (use Fig. n°1 for an illustration). Let me define the function $g(\alpha_A) = \sum_k \Gamma_k \alpha_k^{\sigma-1}$: when $\sigma > 1$ (resp. $\sigma < 1$), I have $\eta_A\eta_B \geq 1 \iff g(\alpha_A) \geq 1$ (resp. $\leq$). For $\sigma \neq 2$, the interior extremum of $g$ is $(\sum_k \Gamma_k^{\frac{1}{\sigma-2}})^{2-\sigma}$, reached for $\alpha_A = \frac{\Gamma_k}{\sum_k \Gamma_k^{\frac{1}{\sigma-2}}}$. For $\sigma = 2$, $g$ is a straight line and has no interior extremum.

$\forall k \in \{A,B\}, x_k = R_k$ is attained for $\alpha$ such that $\frac{1}{\eta} \leq R_k \leq \eta B$. There exists such an $\alpha$ iff $\sum_k \beta_k \Gamma_k^{\frac{1}{\sigma-1}} \leq 1$.

the first condition defines the empty set when $\sigma < 1$ iff $\sum_k \Gamma_k^{\frac{1}{\sigma-1}} > 1$, when $\sigma \in \{1,2\}$ iff $\sum_k \Gamma_k^{\frac{1}{\sigma-1}} < 1$, and when $\sigma > 2$ iff $\forall k, \Gamma_k < 1$. If it does not, it defines a range with bounds $[\alpha_1, \alpha_2]$. $\alpha_{1/2}$ are the (ordered) solutions to equation $g(\alpha) = 1$ when they exist, and respectively 0 and 1 when the corresponding solution does not;

the second condition defines the range $\left[0, \max(\frac{\beta_k}{\Gamma_k^{\frac{1}{\sigma-1}}}, 1)\right]$.

The latter conditions fully characterize feasible investments and are therefore sufficient to prove Lemma n°1.

B  A’s program (proof of Prop. n°1)

Let me first compute the utility derived by $A$ for any given sharing rule $\alpha = (\alpha_A, \alpha_B)$ in the various resulting equilibria.

When the sharing rule induces no investment, the resulting utilities are:

$x_A^* = 0$
$x_B^* = 0$

$\implies \forall k, V_k = \gamma_k R_k$.  \hspace{1cm} (B.1)

When the sharing rule induces full investment, $A$’s resulting utility is:

$V_A(\alpha) = \alpha_A \gamma \left[ \sum_k \mu_k R_k^{\frac{1}{\sigma-1}} \right]^{\frac{1}{\sigma-1}}$

where $V_A$ grows with $\alpha_A$. The binding constraint for $A$ while in that situation is therefore the threshold between $B$ fully and partially investing his asset, in which case:
\[ \alpha_A = 1 - \frac{\beta_B}{\gamma_B} \]
\[ x_A^* = R_A \]
\[ x_B^* = R_B \]
\[
\begin{align*}
V_A &= \gamma \left( \sum_k R_k \frac{\alpha_A^{\sigma - 1}}{\sigma} \right)^{\frac{1}{\sigma - 1}} - \frac{\gamma_B R_B}{\beta_B} \\
V_B &= \frac{\gamma_B R_B}{\beta_B}.
\end{align*}
\]

When the sharing rule induces \( x_A^* = \eta_A R_B < R_A \), A’s resulting utility is:
\[
V_A(\alpha) = \gamma_A R_A + \gamma_B R_B \Gamma_B \frac{\alpha_A^{\sigma/2}}{(1 - \Gamma_A \alpha_A^{\sigma - 1})^{\frac{1}{\sigma - 1}}}.
\]

which grows with \( \alpha_A \). If \( x_A^* = R_A \) is feasible, A always favors it; otherwise, his binding constraint is \( \eta_A R_B \geq 1 \): when \( \sigma < 2 \) he sets \( \alpha = (\alpha_2, 1 - \alpha_2) \); when \( \sigma > 2 \), he set \( \alpha = (\alpha_1, 1 - \alpha_1) \). Using the properties of \( \alpha_{1/2} \), the utilities derived can be written:
\[
\begin{align*}
\alpha_A &= \alpha_{1/2} \\
x_A^* &= \frac{R_A}{\alpha_{1/2}} \\
x_B^* &= R_B
\end{align*}
\]
\[
\begin{align*}
V_A &= \gamma_A R_A + \gamma_B R_B \Gamma_B \frac{\alpha_{1/2}}{(1 - \Gamma_A \alpha_{1/2})^{\frac{1}{\sigma - 1}}} \\
V_B &= \frac{\gamma_B R_B}{\beta_B}.
\end{align*}
\]

Notice that \( x_A^* = R_A \) may be simultaneously feasible; in that case I still have to compare A’s derived utilities in both situations.

When the sharing induces \( x_A^* = R_A \)
\[
\begin{align*}
x_B^* &= \eta_B R_A < R_B \\
x_B^* &= R_B
\end{align*}
\]

\[
V_A(\alpha) = \gamma_A R_A \left( \frac{\alpha_A^{\sigma/2}}{(1 - \Gamma_B \alpha_B^{\sigma - 1})^{\frac{1}{\sigma - 1}}} \right),
\]

which is not always monotonic in \( \alpha_A \) anymore: by increasing his own share, A also reduces B’s investment. Additionally, I have to take into consideration ranges of implementation which are not necessarily convex anymore. \( V_A(\alpha_A) \) has the same sign as \( 1 - \Gamma_B (1 - \alpha_A)^{\sigma - 2}(1 + \alpha_A(\sigma - 1)) \). I need to distinguish several cases.

If \( x_A^* = R_A \)
\[
\begin{align*}
x_B^* &= \eta_B R_A < R_B \\
x_B^* &= R_B
\end{align*}
\]

\[ \sigma < 2 \text{ and } \Gamma_B > 1, \]
\( V_A \) can be defined by extension over \([1 - \frac{1}{\Gamma_B^{\frac{1}{\sigma - 1}}}, 1]\) over which it is decreasing: A would therefore set the minimum \( \alpha_A \) available, thus coming back to the \( x_A^* = R_A \) equilibrium.

If \( x_A^* = R_A \)
\[
\begin{align*}
x_B^* &= \eta_B R_A < R_B \\
x_B^* &= R_B
\end{align*}
\]

\[ \sigma < 2 \text{ and } \Gamma_B < 1, \]
\( V_A \) can be defined by extension over \([0, 1]\). It increases from 0 in 0 until it reaches a maximum, then it decreases until it reaches \( \gamma_A R_A \Gamma_B^{\frac{1}{\sigma - 1}} \) in 1. The investment equilibrium would be implemented for \( \alpha_A \) in the following ranges

\( [\alpha_1, 1] \) when \( \Gamma_A > 1 \) and \( 1 - \frac{\beta_B}{\Gamma_B} = \frac{\beta_A}{\Gamma_A} < \alpha_1 \),

\( [\alpha_1, \alpha_2] \) when \( \Gamma_A < 1 \) and \( 1 - \frac{\beta_B}{\Gamma_B} = \frac{\beta_A}{\Gamma_A} < \alpha_1 \),

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• $[1 - \beta_B \frac{1}{\Gamma_B^A}]$, when $\Gamma_A > 1$ and $\alpha_1 < \beta_A \frac{1}{\Gamma_A^A} < 1 - \beta_B \frac{1}{\Gamma_B^A}$, and

• $[1 - \beta_B \frac{1}{\Gamma_B^A}, \alpha_2]$ when $\Gamma_A < 1$ and $\alpha_1 < \beta_A \frac{1}{\Gamma_A^A} < 1 - \beta_B \frac{1}{\Gamma_B^A} < \alpha_2$.

$V_A'(1 - \beta_B \frac{1}{\Gamma_B^A})$ has the same sign as $1 - \beta_B (\sigma - 1) - \sigma \Gamma_B \beta_B \frac{1}{\Gamma_B^A}$. Therefore if

$$V_B \geq \frac{1 - \beta_B (\sigma - 1)}{\sigma \beta_B \frac{1}{\Gamma_B^A}},$$

$A$ is sure to set $\alpha_A = \alpha_1$ in the two former cases, and to come back to the $x_A^* = R_A$ equilibrium in the two latter. This condition provides an upper limit on the values of $\Gamma_B$ for which $A$ would not find it profitable to induce as much investment from $B$ as possible under the constraint that he invests all his own asset. For $B$ under a value lower than that limit, he is going to offer an intermediate sharing rule, implicitly defined by $\alpha = (\alpha_M, 1 - \alpha_M)$ such that $\Gamma_B (1 - \alpha_M)^{\sigma - 2} (1 + \alpha_M (\sigma - 1)) = 1$.

If $\alpha_1 < 1 - \beta_B \frac{1}{\Gamma_B^A}$ and $\Gamma_B > 1$, $V_A$ can be defined by extension over $[1 - \beta_B \frac{1}{\Gamma_B^A}, 1]$. It decreases first, then increases. Consequently, I need only compare the value of $V_A$ for the lower and upper bound the range of $\alpha$ which implement this equilibrium:

• $[1 - \beta_B \frac{1}{\Gamma_B^A}, \alpha_1]$ when $\Gamma_A < 1$ and $\beta_A \frac{1}{\Gamma_A^A} < 1 - \beta_B \frac{1}{\Gamma_B^A} < \alpha_1$,

• $[1 - \beta_B \frac{1}{\Gamma_B^A}, \alpha_1] \cup [\alpha_2, 1]$ when $\Gamma_A > 1$ and $\frac{\beta_A \frac{1}{\Gamma_A^A}}{1 - \beta_B \frac{1}{\Gamma_B^A}} < 1 - \beta_B \frac{1}{\Gamma_B^A} < \alpha_1$,

• $[1 - \beta_B \frac{1}{\Gamma_B^A}, 1]$ when $\frac{\beta_A \frac{1}{\Gamma_A^A}}{1 - \beta_B \frac{1}{\Gamma_B^A}} < 1 - \beta_B \frac{1}{\Gamma_B^A}$ and $\alpha_1$ and $\alpha_2$, if they exist, are inferior to $\frac{\beta_A \frac{1}{\Gamma_A^A}}{1 - \beta_B \frac{1}{\Gamma_B^A}}$,

• $[\alpha_2, 1]$ when $1 - \beta_B \frac{1}{\Gamma_B^A} < \frac{\beta_A \frac{1}{\Gamma_A^A}}{1 - \beta_B \frac{1}{\Gamma_B^A}} < \alpha_2$.

\[
\frac{V_A(\alpha_1)}{V_A(1 - \beta_B \frac{1}{\Gamma_B^A})} = \frac{\beta_A \frac{1}{\Gamma_A^A}}{1 - \beta_B \frac{1}{\Gamma_B^A}} \Gamma_A^\alpha_1 \Gamma_B^A \]

is lower than 1 in the first range, where consequently $A$ comes back to the $x_A^* = R_A$ equilibrium. In the second range

$$\frac{V_A(1)}{V_A(1 - \beta_B \frac{1}{\Gamma_B^A})} = \frac{1 - \beta_B \frac{1}{\Gamma_B^A}}{\beta_A \frac{1}{\Gamma_A^A}}$$

is higher than 1 for high values of $\sigma$ and $\beta_A$ and for $\Gamma_B < (1 - \beta_B \frac{1}{\Gamma_B^A})^{\sigma - 1}$; if investing $B$’s asset is not profitable enough, $A$ will not involve him in the common endeavor at all; if it is profitable enough, $A$ comes back to the $x_A^* = R_A$ equilibrium. In the third range, the same considerations remain mostly true. In the fourth range, $V_A(\alpha_2) = \Gamma_A^\alpha_2 \alpha_2^2 - 1$, an expression which decreases with $\Gamma_A$ from 1 when $\Gamma_A = 1$. Therefore, in that range, $A$ would prefer $\alpha_A = \alpha_2$. In that last range, $A$ still needs to choose between $\alpha_2$, which leads to $x_A^* = R_A$, $x_B^* = R_B$, and $\alpha_1$, which leads to $x_A^* < R_A$, $x_B^* = R_B$.

The frontier between the two decisions can be implicitly given by:
\[
\frac{\beta_A}{\beta_B} = \frac{\Gamma_A \sigma_1^{-1}}{\Gamma_B (1 - \alpha_2) \sigma_1^{-1} \left( \frac{\Gamma_A \sigma_1^{-1}}{\Gamma_B (1 - \alpha_1) \sigma_1^{-1}} \right)^{\frac{1}{\sigma_1}}}.
\] (B.4)

If \( x_A^* = R_A \)
\[
\begin{align*}
\begin{cases}
x_A^* = R_A \\
x_B^* = \eta_B R_A < R_B
\end{cases}
\end{align*}
\]
\[\sigma > 2 \quad \text{and} \quad \Gamma_B < 1, \quad V_A \text{ is defined over} \quad \alpha_A \in [\max[1 - \frac{\beta_B}{\Gamma_B}, \alpha_2], 1].
\]
\[\Gamma_B \leq \frac{\sigma (\sigma - 1) \sigma_3^{-3}}{\alpha_2}, \quad \text{then} \quad V_A \text{ increases over} [0, 1], \quad \text{and if} \quad \Gamma_B \text{ is under another threshold (higher than the previous one), then} \quad V_A \text{ increases until it reaches a local maximum, decreases until it reaches a local minimum, then increases again until it reaches its absolute maximum in 1: in both cases} \quad A \text{ sets} \quad \alpha_A = 1.
\]
\[\text{Over the latter threshold,} \quad V_A \text{ increases until it reaches its absolute maximum, decreases until it reaches a local minimum, and increases again. At the lower bound of the interval, it is decreasing:} \quad A \text{ has again to choose between} \quad \alpha_2 \quad \text{and} \quad 1, \text{or between} \quad 1 - \frac{\beta_B}{\Gamma_B} \quad \text{and} \quad 1. \quad \text{In each case, the frontier is at the same level.}
\]

The previous analysis uniquely determines all A’s program, and the resulting useful utilities are (I do not present \( \alpha_M \)):

\[
\begin{align*}
\begin{cases}
\alpha_A = \alpha_1/2 \\
x_A^* = R_A \\
x_B^* < R_B
\end{cases}
\end{align*} \iff \begin{align*}
\begin{cases}
V_A = \frac{\gamma_A R_A}{\Gamma_A \alpha_1/2} \\
V_B = \gamma_B R_B + \gamma_A R_A \frac{1 - \alpha_1/2}{\alpha_1/2}
\end{cases}
\] (B.5)
\[
\begin{align*}
\begin{cases}
\alpha_A = 1 \\
x_A^* = R_A \\
x_B^* < R_B
\end{cases}
\end{align*} \iff \begin{align*}
\begin{cases}
V_A = \gamma \mu \frac{\pi}{2} R_A \\
V_B = \gamma B R_B
\end{cases}
\] (B.6)

C First-best asset allocation (proof of Prop. 1°2)

Two properties of the production function are useful: \( Y - \sum_{k} \gamma_k R_k \) is homogeneous of degree 1 and concave (P1), and if \( x_i \neq 0, \lim_{x_i \to 0} \partial_{y_i} f = +\infty \) (P2). If the social planner invests anything, I know that she invests fully at least one asset (P1). Suppose B’s asset is fully invested, then A’s is at least partly invested (P2), and possibly fully invested. I have four situations to consider: no investment, full investment, full investment of B’s asset and partial investment of A’s and conversely.

Let me first determine a condition for the social planner to implement no investment. I turn to polar coordinates: I define \((\rho, \theta)\) such that \( x_A = \rho \cos \theta \) and \( x_B = \rho \sin \theta \), and I consider the slope of \( Y \) along a ray indexed by \( \theta: \)
\[\frac{dY}{d\rho} (\theta) = \gamma (\mu_A \cos(\theta) \frac{d\mu_A}{d\rho} + \mu_B \sin(\theta) \frac{d\mu_B}{d\rho}) \frac{\pi}{2} \]
\[
- \gamma A \cos(\theta) - \gamma B \sin(\theta).
\]
Since \( Y \) is concave (strictly since \( \sigma > 0 \)), and since \( Y'(0) = +\infty \) and \( Y'(\frac{\pi}{2}) = -\infty \), \( \frac{dY}{d\rho} (\theta) \) reaches its maximum in \([0, \frac{\pi}{2}]\). The social planner would not invest in the common endeavor iff this maximum slope along a ray is negative. I only need to determine a condition for this maximum slope to be equal to 0. That condition is that there exists a \( \theta \) for which \( \frac{dY}{d\rho} (\theta) = 0 \) and \( \frac{d^2Y}{d\rho^2} (\theta) = 0 \). I obtain that such a \( \theta \) verifies \( \tan(\theta) = (\frac{x_A \mu_A}{x_B \mu_B})^{\sigma} \). Finally, the social planner invests no asset in the common endeavor iff \( \Gamma_A + \Gamma_B < 1 \).

Notice that for the social planner to invest any asset, she must have found that \( \max_k \frac{\partial_\theta Y}{\partial_\theta x_k} \geq 1 \). Suppose \( \Gamma_i \geq \beta_i \), but \( \Gamma_j < \beta_j \), and suppose \( i \)’s asset is
invested only partially. (P1) ensures that \( j \)'s must be fully invested, which is not possible. Therefore if asset \( i \) is such that \( \Gamma_i \geq \beta_i \), it must be fully invested. The social planner fully invests both assets iff \( \min_k \frac{\Gamma_k}{\beta_k} \geq 1 \). Lastly, consider a situation where \( i \)'s asset is fully invested and \( j \)'s only partially. The social planner maximizes \( Y(x_j) \) and thus invests \( x_j^{FB} = \left( \frac{m_j}{\beta_j} \frac{\Gamma_j}{1-\Gamma_j} \right)^{\frac{1}{\alpha}} R_i. \)

**D Second-best (proof of Prop. n°3)\)**

The social planner is able to reach the first best solution only for \( \sum_k \Gamma_k < 1 \) (she is in fact not at liberty of doing otherwise) and for \( \sum_k \frac{\beta_k}{\Gamma_k} \leq 1 \), in which range \( \begin{cases} x_{A}^{\ast} = R_A \\ x_{B}^{\ast} = R_B \end{cases} \) is feasible and is her desired outcome (in that case, she may have a range of \( \alpha \) leading to it: she has no preference between different sharing rules).

When she is not, two cases may arise. First, if only one non-(0,0) situation is feasible, the social planner chooses \( \alpha \) so as to maximize the investment of the partially invested asset under the condition that the other asset remains invested \( (\eta_{A/B} \geq 1) \). The condition is binding. If \( \sigma > 2 \), \( \eta_{A/B} = 1 \) can only have exactly one solution. Depending on \( \sigma \leq 2 \) and \( A/B \)'s partial investment, that solution is unambiguously \( \alpha_{1/2} \).

Second, \( \begin{cases} x_{A}^{\ast} = R_A \\ x_{B}^{\ast} < R_B \end{cases} \) may be simultaneously feasible while \( \begin{cases} x_{A}^{\ast} = R_A \\ x_{B}^{\ast} = R_B \end{cases} \) is not. This can only happen when \( \sigma > 2, \forall k, \Gamma_k \geq 1 \), and \( g(\alpha) = 1 \) has two solutions. The regulator would induce full investment by \( B \) by choosing \( \alpha \leq \alpha_1 \), and by \( A \) by choosing \( \alpha \geq \alpha_2 \). If she chooses the former, following the same reasoning as before, she would set \( \alpha = \alpha_1 \), and the latter, \( \alpha = \alpha_2 \). Among those two possibilities, she chooses according to which maximizes \( Y \). Equality between the alternative productions provides an implicit equation of the frontier between the two ranges of implementation:

\[
\frac{\beta_A}{\beta_B} = \left( \frac{\Gamma_A \alpha_2^{\sigma-1}}{\Gamma_B (1-\alpha_1)^{\sigma-1}} \right)^{\frac{1}{\alpha}} \frac{1 - \sum_k \Gamma_k \alpha_2^\sigma}{1 - \sum_k \Gamma_k \alpha_1^\sigma} \tag{D.1}
\]

**E Complementarity and the stakes of political competition (proof of Prop. n°4)\)**

I superimpose \( A \)'s program over \( B \)'s. In several situations, there is indeed no conflict, as the best offer that the other can make in his own interest is identical to what one would have offered. There are however several other situations where there is some conflict over the sharing rule. Since the ranges of parameters are unambiguous, I will distinguish the cases by \( \alpha^A \) and \( \alpha^B \). I have seven such situations. I present here the computations for three; the four other cases are: \( \alpha^A = (\alpha_2, 1 - \alpha_2) \) and \( \alpha^B = (0, 1) \); \( \alpha^A = (\alpha_M, 1 - \alpha_M) \) and \( \alpha^B = (\alpha_1, 1 - \alpha_1) \); \( \alpha^A = (1,0) \) and \( \alpha^B = (\alpha_2, 1 - \alpha_2) \); and \( \alpha^A = (1,0) \) and \( \alpha^B = (\beta_A \frac{\Gamma_A}{m_A}, 1 - \beta_A \frac{\Gamma_A}{m_A}) \). I am grateful that the reader exonerates me of the four remaining computations.
As a preliminary, let me examine the evolution of $\alpha_{1,2}$ with respect to $\sigma$. Using the definition of $\alpha$ and the implicit function theorem, I derive that:

$$\frac{d\alpha_1}{d\sigma} = -\frac{\epsilon_i \ln \frac{\epsilon_i}{\mu_i} + (1 - \epsilon_i) \ln \frac{1 - \epsilon_i}{\mu_i}}{(\sigma - 1)^2(\Gamma_i \alpha_i^{\sigma - 2} - \Gamma_j(1 - \alpha_j)^{\sigma - 2})}$$

with $\epsilon_i = \Gamma_i \alpha_i^{\sigma - 1}$. The numerator is always positive, while the denominator is positive either for $\alpha_1$ when $\sigma < 2$ or for $\alpha_2$ when $\sigma > 2$, and negative conversely. As a consequence, when $\sigma > 2$, $\alpha_1$ grows with $\sigma$, while $\alpha_2$ decreases; when $\sigma < 2$, $\alpha_1$ decreases with $\sigma$ while $\alpha_2$ decreases.

**Lemma 2.** The solutions of $g(\alpha) = 1$ are such that:

- when $\sigma > 2$, $\frac{d\alpha_1}{d\sigma} > 0$ and $\frac{d\alpha_2}{d\sigma} < 0$.
- when $\sigma < 2$, $\frac{d\alpha_1}{d\sigma} < 0$ and $\frac{d\alpha_2}{d\sigma} > 0$.

When $\alpha^A = (\alpha_1, 1 - \alpha_1)$ and $\alpha^B = (0, 1)$

$$
\begin{align*}
v_A &= \gamma_B R_B \frac{\alpha_1}{1 - \alpha_1} \\
v_B &= \gamma_B R_B (\Gamma_B - \frac{1}{\mu_B(1 - \alpha_1)^{\sigma - 2}}). 
\end{align*}
$$

(E.1)

Since $\gamma_A$ grows with $\alpha_1$, Lemma n^2 ensures that it grows with $\sigma$. Computations for $\gamma_B$ are a bit more complex:

$$\frac{d\gamma_B}{d\sigma} = \gamma_B R_B \left( -\frac{\ln(\mu_A) \Gamma_B^{\frac{1}{\sigma - 3}}}{(\sigma - 1)^2} - \frac{\epsilon \ln \frac{\epsilon}{\mu} + (2 - \epsilon - \frac{\epsilon}{\alpha_1}) \ln \frac{1 - \epsilon}{\mu}}{(\sigma - 1)(1 - \epsilon)(\Gamma_A \alpha_A^{\sigma - 2} - 1)} \right)$$

The first term inside the parentheses is evidently positive. The second term needs more discussing. Notice first that $\Gamma_A \alpha_A^{\sigma - 2} - 1 = (1 - \alpha_1) \frac{dg(\alpha_1)}{d\sigma} A < 0$. Second, for $\epsilon > \mu_A$, $\ln \frac{1 - \epsilon}{\mu}$ is negative. In that case, $2 - \epsilon - \frac{\epsilon}{\alpha_1} < 2(1 - \epsilon)$. Since $\epsilon \ln \frac{\epsilon}{\mu} + (1 - \epsilon) \ln \frac{1 - \epsilon}{\mu} > 0$ for $\epsilon > \mu_A$, I indeed obtain that in that case, $\frac{dg}{d\sigma} B > 0$. Third, for $\epsilon < \mu_A$, using Lemma n^2, I can rewrite $\frac{dg}{d\sigma}$ as the sum of three positive terms:

$$\frac{d\gamma_B}{d\sigma} = \gamma_B R_B \left( -\frac{\ln(\mu_A) \Gamma_B^{\frac{1}{\sigma - 3}}}{(\sigma - 1)^2} - \frac{\epsilon \ln \frac{\epsilon}{\mu} + (1 - \epsilon) \ln \frac{1 - \epsilon}{\mu}}{(\sigma - 1)(1 - \epsilon)(\Gamma_A \alpha_A^{\sigma - 2} - 1)} + \frac{\ln \frac{1 - \epsilon}{\mu}}{(\sigma - 1)(1 - \epsilon)} \right).$$

As a conclusion, higher complementarity of assets makes it less valuable for both $A$ and $B$ to be in power in that particular case, as in the following ones. When $\alpha^A = (1 - \frac{\beta}{\mu} \frac{1}{\sigma - 3}, \frac{\beta}{\mu} \frac{1}{\sigma - 3})$ and $\alpha^B = (0, 1)$

$$
\begin{align*}
v_A &= \gamma \left( \sum_k \mu_k R_k^{\frac{1}{\sigma - 1}} \right)^{\frac{1}{\sigma - 1}} \left( 1 - \frac{\beta}{\mu} \frac{1}{\sigma - 1} \right) - \gamma A R_A \\
v_B &= \gamma \left( \sum_k \mu_k R_k^{\frac{1}{\sigma - 1}} \right)^{\frac{1}{\sigma - 1}} \left( \beta \frac{1}{\sigma - 1} \frac{\gamma}{\mu} \frac{1}{\sigma - 1} \right) - \frac{\beta}{\mu} \frac{1}{\sigma - 1} 
\end{align*}
$$

(E.2)

$$\begin{align*}
\frac{dv_A}{d\sigma} &= \gamma \left( \sum_k \mu_k R_k^{\frac{1}{\sigma - 1}} \right)^{\frac{1}{\sigma - 1}} \sum_k \beta_k \ln \frac{\beta_k}{\mu_k} - \frac{\beta}{\mu} \frac{1}{\sigma - 1} \ln \frac{\mu}{\mu} \\
\frac{dv_B}{d\sigma} &= -\gamma A R_A \frac{\ln(\mu_A)}{(\sigma - 1)^2} - \frac{\beta}{\mu} \frac{1}{\sigma - 1} \ln \frac{\mu}{\mu} 
\end{align*}$$

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Both expressions are the sum of two positive terms on the relevant ranges of parameters, and are thus positive themselves.

When \( \alpha^A = (1 - \frac{\beta_A}{1 - \gamma_A}, \frac{\beta_A}{1 - \gamma_A}) \) and \( \alpha^B = (\frac{\beta_A}{1 - \gamma_A}, 1 - \frac{\beta_A}{1 - \gamma_A}) \)

\[
v_A = v_B = \gamma \left( \sum_k \mu_k R_k^{\frac{\sigma - 1}{\sigma}} \right)^{\frac{\sigma}{\sigma - 1}} - \sum_k \frac{\gamma_k R_k}{\beta_k} \]

\[
= \gamma \left( \sum_k \mu_k R_k^{\frac{\sigma - 1}{\sigma}} \right)^{\frac{\sigma}{\sigma - 1}} \left( 1 - \sum_k \frac{\beta_k}{\mu_k} \right)
\]

(E.3)

\[
dv_A \over d\sigma = dv_B \over d\sigma = \frac{\gamma \left( \sum_k \mu_k R_k^{\frac{\sigma - 1}{\sigma}} \right)^{\frac{\sigma}{\sigma - 1}}}{(\sigma - 1)^2} \sum_k \zeta_k \ln \frac{\beta_k}{\mu_k}
\]

with \( \zeta_i = \beta_i + \frac{\sigma - 1}{\sigma} \left( \beta_i \frac{1}{\mu_i} - \beta_i \frac{1}{\gamma_A \mu_i} \right) \). Notice that \( \ln \frac{\beta_i}{\mu_i} \) and \( \frac{\beta_i}{\mu_i} \) are of opposite sign. Since \( \zeta_i \geq \beta_i \), if \( \beta_i \geq \mu_i \)

\[
dv_A \over d\sigma \geq \frac{\gamma \left( \sum_k \mu_k R_k^{\frac{\sigma - 1}{\sigma}} \right)^{\frac{\sigma}{\sigma - 1}}}{(\sigma - 1)^2} \sum_k \beta_k \ln \frac{\beta_k}{\mu_k} \geq 0.
\]

The problem being symmetric between \( A \) and \( B \), I extend this result to any \( \beta \) and \( \mu \): \( dv_A \over d\sigma = dv_B \over d\sigma \geq 0 \).

\section*{F Commitment and complementarity (proof of Prop. n°6)}

\( W_A \) increases in \( p \) over \( [0, \alpha_A] \) and over \([\overline{p}, 1] \). Both \( V_A^r \) and \( C \) decrease in \( p \) over \([\alpha_A]^R, \overline{p}(\sigma)] \). Therefore \( p_A^* \) \( \in [\alpha_A^R, \overline{p}] \cup \{1\} \), as indicated in the main text.

There are four possibilities for any given \( \sigma \): first, that it has a neighborhood in which \( p_A^*(\sigma) = \alpha_A^R(\sigma) \) (the maximum remains locally stuck at the lower bound); second, that it has a neighborhood in which \( p_A^*(\sigma) = 1 \) (the maximum remains locally stuck at 1); third, that \( A \) is indifferent between setting \( p_A^* = 1 \) and another value in \([\alpha_A^R(\sigma), \overline{p}(\sigma)] \); and fourth is the interior situation.

In the first possibility, \( A \) finds that the gains from commitment outweigh the cost up to his optimal offer of sharing rule. Prop. n°6 is locally equivalent to his optimal offer increasing with complementarity, in other words, to \( \frac{d\alpha_r}{d\sigma} \geq 0 \).

From Prop. n°1, \( \alpha_A^n \) can take four nontrivial values: \( 1 - \frac{\beta_A}{\mu_A} \frac{1}{\gamma_A}, \alpha_M \) and \( \alpha_{1/2} \).

Lemma n°2 and simple computations show that these values increase with \( \sigma \).

In the second possibility, Prop. n°6 is locally trivial.

I cover the two latter possibilities by making an argument in the line of Topkis’s theorem. To finish proving Prop. n°6, I need only show that in the noncorner case and to its right, \( W_A \) is submodular, or equivalently, that \( \forall p \geq \alpha_A^* \),

\[
\frac{d^2 W_A(p, \sigma)}{dp d\sigma} = \frac{d^2 V_A^r(p, \sigma)}{dp d\sigma} \leq 0.
\]

The noncorner case can never implement \( x_n^* < R_A \) or \( x_B^* = R_B \) or \( x_B^* = R_B \), since when these situations are feasible, I have
shown in the proof of Lemma n°1 that should A implement either, he would always choose $\alpha_A^+$ at the upper bound of the implementability range. Therefore $p$’s constraint can implement a noncorner $\left\{ \begin{array}{l}
 x_A^* = R_A \\
 x_B^* < R_B 
\end{array} \right.$ from which A derives utility $\frac{p \gamma R_A \mu_A}{(1 - \frac{\gamma (1-p)^{\sigma-1}}{\gamma B} \mu_B)^{\frac{\sigma}{\sigma - 1}}}$. Writing $x = \frac{\gamma (1-p)^{\sigma-1}}{\gamma B} \mu_B$, its cross-derivative is

$$\frac{\gamma R_A \mu_A}{(\sigma-1)^{\frac{\sigma}{\sigma - 1}}} \left[ \left( \frac{1 - \sigma px}{(1-p)(1-x)} \right) \ln \frac{1-x}{\mu_A} + \frac{\sigma x}{1-x} \left( 1 - \frac{\sigma px + (\sigma-1) p}{(1-p)(1-x)} \right) \ln \frac{x}{\mu_B} - \frac{\sigma(\sigma-1)^{\sigma} px}{(1-p)(1-x)} \right].$$

The second factor of this expression can be written as the sum of negative terms when $p \geq \alpha_A^+$.

Therefore, in all four possibilities, when $\sigma$ increases, $p_A^*$ either increases continuously or it “jumps” to an greater value, which concludes the proof of Prop. n°6.